
Every year, the National Science Foundation supports several CBMS-NSF Regional Research Conferences to stimulate interest in a wide range of topics of mathematical research. Each conference features a distinguished lecturer who delivers ten lectures on a topic of current interest focused on a particular area of the mathematical sciences. Subsequently, the lecturer prepares an expository monograph based upon the lectures. The book under review belongs to this series and is the result of the lectures delivered by Adrian Constantin in 2010. I personally own a number of books from the CBMS-NSF series and have eagerly awaited the release of the current book under review, having attended the lectures myself.

The book is titled and styled after the lecture series. It does, however, contain significantly more detail than that presented in the original lectures. The additional material is certainly welcome as it serves to emphasize the expository intent of the CBMS-NSF series. The book is intended for mathematicians, physicists, and engineers interested in water waves and stresses the interplay between mathematical ideas and physical insight. This book does not shy away from technicalities and several passages discuss mathematical details at length, though there is a consistent attempt to relate the insight gained from theorems to physical experiments and actual measurements of water waves. Each chapter is followed by a meticulously researched appendix highlighting the mathematical techniques and ideas underlying the analysis introduced in that chapter.

Of course, the subject of water waves being so well studied, Professor Constantin makes no claim to an exhaustive survey of the entire subject. Indeed the book clearly states the intended topics of discussion:

- **Wave-current interaction.** The main goal here is the existence theory for two-dimensional steady water waves propagating over a flat bed in a flow with a general vorticity distribution. Typically, a current is modeled as a steady shear flow, i.e., a horizontal flow varying in the vertical direction. The gradient of the flow introduces vorticity into the problem.

- **Flow pattern beneath irrotational waves.** Various properties of two-dimensional flow (gradients of the pressure and fluid velocities, particle paths, etc.) are discussed for steady irrotational waves on the surface of water over a flat bed.

- **Wave breaking.** One of the characteristic features of water waves is the phenomenon of breaking, producing some of the most elegant and breathtaking patterns in nature. This book describes some models (one space- and one time-variable) of wave breaking to provide insight into this unyielding area of mathematical research.

- **Application of water wave theory to tsunamis.** The relevance of tsunami research does not need stating. In the final topic some aspects of tsunami modeling are discussed.

It is worth noting the topics not discussed in this book. This book does not discuss viscous effects for water waves. Indeed, using scaling arguments and actual numbers relevant to the oceans, the case is made that viscous effects are unimportant for the topics under discussion. In a similar vein, surface tension effects are not discussed. The presence of turbulence in the flow field creates additional difficulty. Indeed, the mathematical analysis is significantly more challenging, to say the least, and as a result, the discussion here is limited to nonturbulent fluid flows. Last, in keeping with the analytical bent of the book, barring few exceptions (Chapter 3 presents numerical simulations of large amplitude rotational waves), numerical techniques and solutions are not emphasized.

Chapter 3 forms the bulk of the book and describes wave-current interactions. Following the usual approach for two-
dimensional fluid flow problems, a stream function formulation is employed to convert the problem of existence of periodic traveling wave solutions to a free boundary-value problem for an elliptic partial differential equation. The book makes extensive use of bifurcation theory to establish existence of a continuum of both small and large amplitude waves. The main result is the existence of a continuum of solutions up to a wave containing a stagnation point, i.e., a point where the local fluid velocity equals the speed of the wave. The appendix to Chapter 3 contains a terse introduction to degree theory in finite and infinite dimensions, Crandall–Rabinowitz bifurcation theory, and analytic global bifurcation with plenty of references, should the interested reader be inclined to read more on these topics. They are clearly explained here, but perhaps are not suitable as an introduction to the subject.

The following chapter discusses several fluid properties such as the gradient of the pressure and fluid velocities, the shape of a traveling water wave, etc. The discussion is limited to irrotational flows. I thoroughly enjoyed this chapter, which contains a delightful application of the Hopf maximum principle to deduce monotonicity properties of pressure and fluid velocity. Many of the results confirm our intuitive understanding; however, such results have never before been rigorously established.

After the extensive discussion of periodic waves, we turn our attention in Chapter 5 to solitary waves, i.e., traveling surface water waves on the whole real line. We begin with the obligatory recounting of the tale of John Scott Russell and “the great wave of translation” from his observation in 1834. Some reference is made to solitary waves with vorticity, but the main results of this chapter are confined to irrotational flows. The focus in this chapter is on the motion of fluid particles and the fluid pressure beneath an irrotational solitary water wave. The discussion of the pressure is analogous to that of Chapter 4 and hence is qualitative. The chapter ends with a brief passage on solitons and soliton interactions. This chapter contains the largest appendix in the book (almost four times the size of the chapter itself). The appendix covers a wide range of topics from integrable systems including Hamiltonian dynamics, symmetries and conservation laws, the isospectral problem, and inverse scattering. This appendix also provides a quick introduction to Riemann–Hilbert problems and the algebraic-geometric approach to integrability of partial differential equations such as Korteweg–deVries (KdV). If nothing else, the extensive appendix to this chapter should convince the reader of the vast range of mathematical tools used in the study of water waves.

Our first departure from the full set of Euler equations describing fluid flow coincides with our first analysis of time-dependent problems, in particular the problem of wave breaking. Wave breaking is modeled as a kind of blow-up, specifically in the spatial derivative of the surface profile, and so we require model equations which permit such blow-up in finite time. The reader is introduced to one such model equation: the Johnson equation. This equation can be obtained as a small to moderate amplitude approximation of Euler’s equation for fluid flow. Interestingly, the equations for the surface profile and surface velocity are not identical, unlike the more well-known KdV regime.

The final chapter presents a change of pace and style. The full tsunami problem is considerable and the reader is forced to reconsider the problem in a more ideal light. This book largely focuses on fluid flows over flat beds, whereas bottom topography plays an important role in tsunami modeling, especially near the coastline. In the absence of mathematical generality as well as analytical tools, Professor Constantin ponders the simple and basic question of whether the KdV equation is relevant to the modeling of tsunamis, particularly in the open ocean. After an analysis of the length scales involved, we are presented with estimates of the distance a wave will travel before the balance between nonlinearities and dispersion can occur. For a wave generated in an ocean of 4 km depth, the wave would need to travel thousands of kilometers before the KdV balance is achieved. Consequently, Professor Constantin concludes that the KdV equation is not relevant for tsunami modeling. Following this, the reader is presented with some numbers from two tsunami events in the recent history:
the 2004 Pacific Ocean tsunami and the 1960 Chilean tsunami. Both incidents re-emphasize the conclusion that linear wave theory provides a good model of tsunami propagation in the open ocean.

In summary, I think this book is a welcome addition to the body of literature on water waves. It is a handy compilation of various aspects of the water wave problem and a good survey of different mathematical techniques. I particularly appreciated the individual appendices to each chapter, adding prerequisite material where required and pointing the reader to a reference when appropriate. The book is on the more mathematical side and the mathematically mature reader will gain most from it. This is perhaps inevitable with the mathematical investigation of water waves, which is a vast subject, and I am skeptical that any one book could satisfy the need of every researcher and student. Nonetheless, I do recommend this book to anyone interested in the theory of water waves and indeed to any student of applied analysis.

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