Preface

This book is an expanded version of the 10 CBMS lectures I delivered at Tulane University in May, 2010.

The main theme of this book is diffusion: from Turing's "diffusion-driven instability" in pattern formation to the interactions between diffusion and spatial heterogeneity in mathematical ecology. Along the way we will also discuss the effects of different boundary conditions—in particular, those of Dirichlet and Neumann boundary conditions.

On the dynamics aspect, in Chapters 1 and 2, we will start with the fundamental question of stabilization of solutions, including the rate of convergence. It seems interesting to note that the geometry of the underlying domain, although still far from being fully understood, plays a subtle role here.

It is well known that steady states play important roles in the dynamics of solutions to parabolic equations. In Chapter 3 we will focus on the qualitative properties of steady states, in particular, the "shape" of steady states and how it is related to the stability properties of steady states. This chapter is an updated version of relevant materials that appeared in an earlier survey article [N3].

In the second half of this book, we will first explore the interactions between diffusion and spatial heterogeneity, following the interesting theory developed mainly by Cantrell, Cosner, Lou, and others in mathematical ecology. Here it seems remarkable to note that even in the classical Lotka–Volterra competition-diffusion systems, the interaction of diffusion and spatial heterogeneity creates surprisingly different phenomena than its homogeneous counterpart. This is described in Chapter 4. Finally, in Chapter 5, we include several models beyond the usual diffusion, namely, various directed movements including chemotaxis and cross-diffusion models in population dynamics. This direction seems significant—from both modeling and mathematical points of view—one step into more sophisticated and realistic modeling with challenging and significant mathematical issues. In this regard, we would like to recommend the recent article [L3] to interested readers for a more thorough and complete survey.

Diffusion has been used extensively in many disciplines in science to model a wide variety of phenomena. Here we have included only a small number of models to illustrate the depth and breadth of the mathematics involved. The selection of the materials included here depends solely on my taste—not to reflect value judgement. One unfortunate omission is traveling waves, especially those with curved fronts. Interested readers are referred to the recent survey article of Taniguchi [Tn].

I wish to take this opportunity to thank the organizer of this CBMS conference, Xuefeng Wang, my old friend, Morris Kalka, and my colleagues and staffs at Tulane University for organizing this wonderful conference. I also wish to thank all the participants, some of

whom came from faraway places, including China, Hong Kong, Japan, Korea, and Taiwan. It is a true pleasure to express my sincere appreciation to the five one-hour speakers, Chris Cosner, Manuel del Pino, Changfeng Gui, Kening Lu, and Juncheng Wei, for their inspiring lectures.

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