Preface

The numerical approximation of stochastic partial differential equations (SPDEs), specifically, stochastic evolution equations of the parabolic or hyperbolic type, encounters all of the difficulties that arise in the numerical solution of both deterministic PDEs and finite dimensional stochastic ordinary differential equations (SODEs) as well as many more due to the infinite dimensional nature of the driving noise processes. The state of development of numerical schemes for SPDEs compares with that for SODEs in the early 1970s. Most of the numerical schemes that have been proposed to date have a low order of convergence, especially in terms of an overall computational effort, and only recently has it been shown how to construct higher order schemes.

The breakthrough for SODEs started with the Milstein scheme and continued with the systematic derivation of stochastic Taylor expansions and the numerical schemes based on them. These stochastic Taylor schemes are based on an iterated application of the Itô formula. The crucial point is that the multiple stochastic integrals which they contain provide more information about the noise processes within discretization subintervals, and this allows an approximation of higher order to be obtained. This theory is presented in detail in the monographs Kloeden & Platen [82] and Milstein [95].

There is, however, no such Itô formula for the solutions of stochastic PDEs in Hilbert spaces or Banach spaces (see Chapter 7 for more details). Nevertheless, it has recently been shown that Taylor expansions for the solutions of such equations can be constructed by taking advantage of the mild form representation of the solutions. Moreover, such expansions are robust with respect to the noise in the additive noise case, i.e., hold for other types of stochastic processes with Hölder continuous paths such as fractional Brownian motion.

This book is based on recent work of the coauthors. Its style, contents, and structure follow the series of lectures given by the second author, Peter Kloeden, in August 2010 at the Illinois Institute of Technology in Chicago. The main difference from the lectures is the existence and uniqueness theorem in Chapter 5. Most of that chapter and the entire appendix were written by the first coauthor, Arnulf Jentzen.
The book also includes new developments on numerical methods for random ordinary differential equations and SODEs, since these are relevant for solving spatially discretized SPDEs as well as in their own right. The focus is on pathwise and strong convergences. In finance mathematics, weak convergence is of primary interest, but strong convergence is nevertheless important, too, as an essential component of the multilevel Monte Carlo method introduced recently in Giles [35] (see also Heinrich [54]).

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Arnulf Jentzen, Princeton

Peter Kloeden, Frankfurt am Main