Preface

In the past thirty years the field of inverse scattering theory has become a major theme of applied mathematics with applications to such diverse areas as medical imaging, geophysical exploration, and nondestructive testing. The growth of this field has been characterized by the realization that the inverse scattering problem is both nonlinear and ill-posed, thus presenting particular problems in the development of efficient inversion algorithms. Although linearized models continue to play an important role in many applications, the increased need to focus on problems in which multiple scattering effects can no longer be ignored has led to the nonlinearity of the inverse scattering problem playing a central role. In addition, the possibility of collecting large amounts of data over limited regions of space has led to the situation where the ill-posed nature of the inverse scattering problem becomes of central importance.

Initial efforts to deal with the nonlinear and ill-posed nature of the inverse scattering problem focused on the use of nonlinear optimization methods, in particular Newton's method and various versions of what are now called decomposition methods. For a discussion of this approach to the inverse scattering problem together with numerous references, we refer the reader to [42]. Although efficient in many situations, the use of nonlinear optimization methods suffers from the need for strong a priori information in order to implement such an approach. Hence, in order to circumvent this difficulty, a recent trend in inverse scattering theory has focused on the development of a qualitative approach in which the amount of a priori information needed is drastically reduced but at the expense of obtaining only limited information of the scatterer such as the connectivity, support, and an estimate on the values of the constitutive parameters. Examples of such an approach are the linear sampling method, the factorization method, and the theory of transmission eigenvalues. It is these topics that are the theme of this book, focusing on their use in the inverse acoustic scattering problem for inhomogeneous media.

The qualitative approach to inverse scattering theory was initiated by Colton and Kirsch in 1996 [39]. In this paper they introduced a linear integral equation of the first kind, called the far field equation, whose solution could be used as an indicator function to determine the support of the scattering obstacle. This method is called the linear sampling method. The mathematical difficulties inherent in this approach were subsequently resolved by the factorization method of Kirsch [74], and further clarification of the relationship between the linear sampling and factorization methods was obtained by Arens and Lechleiter [4] and Audibert and Haddar [7]. Having determined the support of the scatterer, the next step in the qualitative approach is to determine estimates on the material properties of the scatterer. This was accomplished by Cakoni, Gintides, and Haddar [27] through the use of transmission eigenvalues first introduced by Kirsch [73] and Colton and Monk [46]. The development of the above themes is the subject matter of the chapters that follow. This book is intended for mathematicians, physicists, and engineers who are either actively involved in problems arising in scattering theory or have an interest in this field and wish to know more about recent developments in this area. It will also be of interest to advanced graduate students wishing to become more informed about new ideas in inverse scattering theory. On the other hand, for those unfamiliar with classical scattering theory, Chapter 1 provides a basic introduction to this area and also serves as an introduction to the chapters which follow.

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