

# Index

*Boldfaced type indicates terms of particular importance  
and page numbers on which terms are defined.*

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