Preface

Methods for the numerical simulation of dynamic mathematical models have been the focus of intensive research for well over 60 years. However, rather than reaching closure, there is a continuing demand today for better and more efficient methods, as the range of applications is increasing. Mathematical models involving evolutionary partial differential equations (PDEs) as well as ordinary differential equations (ODEs) arise in many diverse applications, such as fluid flow, image processing and computer vision, physics-based animation, mechanical systems, relativity, earth sciences, and mathematical finance. It is possible today to dream of, if not actually achieve, a realistic simulation of a clothed, animated figure in a video game, or of an accurate simulation of a fluid flowing in a complex geometry in three dimensions, or of simulating the dynamics of a large molecular structure for a realistic time interval without requiring several weeks of intense computing.

This text was developed from course notes written for graduate courses that I have taught repeatedly over the years. The students typically come from different disciplines, including Computer Science, Mathematics, Physics, Earth and Ocean Sciences, and a variety of Engineering disciplines. With the widening scope of practical applications comes a widened scope of an interested audience. This means not only varied background and expertise in a typical graduate class, but also that not all those who need to know how to simulate such PDE systems are or should be experts in fluid dynamics! The approach therefore chosen emphasizes the study of principles, properties, and usage of numerical methods from the point of view of general applicability. This text is not a collection of recipes, and basic analysis and understanding are emphasized throughout, yet a formal theorem–proof format is avoided and no topic is covered simply for its theoretical beauty. Moreover, while no one class of applications motivates the exposition, many examples from different application areas are discussed throughout.

In addition to not relying on strict fluid dynamics prerequisites, the other strategic decision made was to delay in each chapter as much as possible the separation of treatment of parabolic and hyperbolic equations. The other route, taken by many authors, is to devote separate chapters for the different PDE types. This often leads to a very neat presentation. In this text the approach is more concept oriented, however, and it is hoped that the differences necessarily highlighted by contrasting the treatments in this way shed more direct light on some issues. Moreover, questions about problems such as simulating a convection-dominated diffusion-convection process and about mixed hyperbolic-parabolic systems are more naturally addressed.

The introductory Chapter 1 is essential. First we develop a sense for the types of mathematical PDE models for which solving evolutionary problems makes sense, by studying well-posedness. Then we embark upon an introduction by example to numerical methods and issues that are developed more fully later on.

Several years ago L. Petzold and I wrote a book in a similar spirit about the numerical solution of ODEs. I still like that work, and thankfully so do others; however, there are many students and researchers in various disciplines who simply don't seem to have room in their program to accommodate a course or a text devoted solely to numerical ODEs. Therefore, I have included in the present text two chapters on this topic. Chapter 2 crams in all the material from our book [14] that is viewed as essential for a crash course on simulating ODEs, with an eye toward what is essential and relevant for PDEs. Chapter 6, not covered in [14], is more specialized and considers certain problems and methods in Geometric Integration, especially for Hamiltonian systems. These concepts and methods are relevant also for PDEs—see especially Chapter 7—and they allow a different and interesting look at numerical methods for differential equations. However, they are less essential in a way.

Chapter 3 develops in detail the basic concepts, issues, and discretization tools that arise in finite difference and finite volume methods for PDEs. It relates to and expands the material in the previous two chapters but they are not a strict prerequisite for reading it.

Stability is an essential concept when designing and analyzing methods for the numerical solution of time-dependent problems. Chapter 4 deals with constant coefficient PDEs where resulting criteria are relatively easy to check. Chapter 5 continues into variable coefficient and nonlinear problems, where stability criteria and approaches are defined and used, and where new methods for hyperbolic PDEs are introduced.

The first five chapters are the basic ones, and it is reasonable to teach in a semester course mainly these plus some forays into later chapters, e.g., Chapter 6 if one concentrates significantly on ODEs. On the other hand, many more delicate or involved issues in numerical PDEs, and much of the more recent research, are in the last five chapters. The topic of numerical dispersion in wave problems and of conservative vs. dissipative methods is considered in Chapter 7, while handling solutions with discontinuities is discussed at some length in Chapter 10. These chapters are both somewhat more topical and occasionally more advanced, and neither is a prerequisite for the other. The handling of boundary conditions is briefly considered in Chapter 3, but Chapter 8 discusses deeper and more specialized issues, particularly for hyperbolic PDEs. Two related but different topics are considered in Chapter 9. The first concerns handling additional issues that arise for problems in more than one space variable. There are many issues here, and they are necessarily covered occasionally in less depth than would be possible in a more specialized monograph. I have used this chapter also to introduce and discuss the interesting class of splitting methods, even though these arise not only in multidimensional problems. Finally, Chapter 11 quickly describes some highly interesting related topics that could require separate monographs if they were to be fully treated.

There are several reviews of background material collected in separate sections in Chapters 1 and 2. These are meant as refreshers to quickly help a reader who has been exposed to their contents beforehand, not to replace a proper introduction to their subject matters. The survey of iterative methods for linear systems of algebraic equations in Section 9.4 is somewhere between a review and a core material. At the end of each chapter there are also exercises, the first of which (numbered 0) consisting of more straightforward review questions. I have tried to indicate those exercises that have been found to be more difficult, or more time-consuming, among them.

No attempt has been made to make the bibliography complete: this would have been a vast and dangerous undertaking in itself. Wherever possible I have tried to refer to texts, monographs, and survey articles that contain a wealth of additional references. Several references to my own work are there simply because I've naturally drawn upon past work as a source for examples and exercises and because I am generally more familiar with them than with others.

Parts of Chapters 2, 6, and 7 were written for a short course that I gave at the Institute of Pure and Applied Mathematics (IMPA) in Rio de Janeiro. Here is an opportunity to thank my friends and colleagues there, especially D. Marchesin, A. Nachbin, M. Sarkis, L. Velho, and J. Zubelli, for their hospitality during several visits to the "marvelous city." I also gratefully acknowledge the hospitality of A. Iserles and the pleasant Isaac Newton Institute in Cambridge, where much of the material presented here was polished during a seven-week stay last year.

Many people have helped me in various ways to shape, reshape, refine, and debug this text. Several generations of our students had to endure its (and my) imperfections and their various comments have helped tremendously. In particular let me note Eddy Boxerman, Hui Huang, and Ewout van den Berg; there are many others not mentioned here. Colleagues who have read various versions of these notes and/or offered loads of advice and useful criticism include in particular Mark Ainsworth, Mihai Anitescu, Evaristo Biscaia Jr., Robert Bridson, Chris Budd, Philippe Chartier, Chen Greif, Eldad Haber, Ernst Hairer, Arieh Iserles, Robert McLachlan, Sarah Mitchell, Dinesh Pai, Linda Petzold, Ray Spiteri, David Tranah, and Jim Varah. I am indebted to you all!

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