Preface

This book provides an introduction to some modern computational techniques for optimization problems governed by partial differential equations (PDEs). The optimization framework used is that of optimization on functional spaces based on the Lagrange formalism. The computational techniques discussed in this book represent recent developments resulting mainly from the combination of modern techniques for the numerical solution of PDEs and of sophisticated optimization schemes. Based on these two main features, the book provides a bridge between continuous optimization and PDE modeling focusing on the numerical solution of the corresponding problems.

Although the topics considered in this book are advanced, the methods and problems are presented in much detail such that the book may serve as a textbook for graduate students, requiring previous knowledge of the reader on the numerical solution of PDEs and on basic principles of continuous optimization. The book should also be suitable as an introduction for researchers in the field of scientific computing with PDEs to work in the field of optimization and for researchers in optimization to consider methodologies coming from the field of numerical PDEs. It should also allow scientists of natural sciences and engineering to formulate and solve optimization problems appropriate for their research purposes.

This book is timely considering the increasing interest in the field of PDE optimization, as can be seen from the many recent conferences and workshops worldwide and from the enormous increase in publications in this field. The focus of the book is on methodological aspects, and therefore applications are less emphasized. Problems with elliptic, parabolic, and hyperbolic PDEs and coupled systems of PDEs are considered.

It is clear that the purpose of this book is too ambitious to succeed completely. However, it fills a gap between more theoretically oriented texts on PDE constrained optimization and algorithmic oriented books. Furthermore, the intention to cover many emerging topics in numerical PDE optimization obviously results in less background material for which many references are provided. Nevertheless, enough details are devoted to each topic to understand the formulation of the problems and of the proposed solution procedures such that this book could be used as an introductory textbook for some very recent research topics of PDE optimization. It is obvious that the choice of these topics is biased by the research interests and experience of the authors, and this fact explains some focusing on specific topics rather than others.

This book is organized as follows. After a brief introduction, we start discussing the theoretical framework of PDE optimization. We focus on the characterization of the solution of optimization problems by means of the so-called optimality systems. These are coupled systems of PDEs and differential or scalar equalities or inequalities called the

optimality conditions. It should be remarked that these systems have very special structures posing new challenges for the scientific computing methodologies. This challenge starts with the appropriate discretization of optimality systems that is the subject of one of the chapters of this book. Once the PDE optimization problem is discretized, a solution-optimization procedure can be started. One chapter is devoted to single-grid optimization schemes that should refer to the optimization procedures that do not exploit the possibility to represent the given problem on multiple scales of resolution. On the contrary, a chapter on multigrid methods illustrates recent advances that combine classical optimization schemes with the multigrid strategy. Another chapter is dedicated to recent methodologies that combine PDE optimization with statistical approaches in order to solve PDE optimization problems with uncertainty. Finally, a chapter of applications is presented with the purpose of outlining important emerging application topics such as shape optimization, quantum control, and time-dependent electromagnetic inverse problems. Summarizing, we have the following chapters:

1. Introduction
2. Optimality Conditions
3. Discretization of Optimality Systems
4. Single-Grid Optimization
5. Multigrid Methods
6. PDE Optimization with Uncertainty
7. Applications

We would like to thank many friends who encouraged this work and contributed through remarks, suggestions, and scientific collaboration. In particular, we would like to acknowledge the support and collaboration of (in alphabetical order) Mario Annunziato, Hans Georg Bock, Giuseppe Borzì, Nicolas Gauger, Omar Ghattas, Sergio Gonzalez Andrade, Matthias Heinkenschloss, Roland Herzog, Ulrich Hohenester, Kazufumi Ito, Do Young Kwak, Karl Kunisch, Roberta Mancini, Stephen Nash, Hans Josef Pesch, Ekkehard Sachs, Julien Salomon, Stephan Schmidt, Claudia Schillings, Georg Stadler, Fredi Tröltzsch, Michelle Vallejos, Stefan Volkwein, and Greg von Winckel.

We would like to thank Ms. Elizabeth Greenspan of SIAM Publications very much for her kind and generous assistance in publishing this book. Furthermore, we would like to thank an unknown referee, who helped to improve the final version of this book significantly.

We hope that this book will become a useful tool to everyday research and teaching work and that it comes to further editions because “Perfection is achieved, not when there is nothing more to add, but when there is nothing left to take away” (Antoine de Saint-Exupéry).