

Contents

| | |
|---|-----------|
| Preface | xi |
| I Linear Inverse Problems | 1 |
| 1 Introduction | 3 |
| 2 Naïve reconstructions and inverse crimes | 7 |
| 2.1 Convolution | 7 |
| 2.2 Heat propagation | 14 |
| 2.3 Tomographic X-ray projection data | 21 |
| 3 Ill-posedness in inverse problems | 35 |
| 3.1 Forward map and Hadamard's conditions | 35 |
| 3.2 Ill-posedness of the backward heat equation | 36 |
| 3.3 Ill-posedness in the continuous case | 40 |
| 3.4 Regularized inversion | 47 |
| 3.5 The SVD for matrices | 49 |
| 3.6 SVD for the guiding examples | 51 |
| 4 Truncated singular value decomposition | 53 |
| 4.1 Minimum norm solution | 53 |
| 4.2 Truncated SVD | 55 |
| 4.3 Measuring the quality of reconstructions | 56 |
| 4.4 TSVD for the guiding examples | 57 |
| 5 Tikhonov regularization | 63 |
| 5.1 Classical Tikhonov regularization | 63 |
| 5.2 Normal equations and stacked form | 67 |
| 5.3 Generalized Tikhonov regularization | 69 |
| 5.4 Choosing the regularization parameter | 72 |
| 5.5 Large-scale implementation | 78 |
| 6 Total variation regularization | 83 |
| 6.1 What is total variation? | 83 |
| 6.2 Quadratic programming | 86 |

| | | |
|-----------|---|------------|
| 6.3 | Sparsity-based parameter choice | 88 |
| 6.4 | Large-scale implementation | 90 |
| 7 | Besov space regularization using wavelets | 95 |
| 7.1 | An introduction to wavelets | 95 |
| 7.2 | Besov spaces and wavelets | 98 |
| 7.3 | Using B_{11}^1 regularization to promote sparsity | 99 |
| 8 | Discretization-invariance | 103 |
| 8.1 | Tikhonov regularization and discretizations | 104 |
| 8.2 | Total variation regularization and discretizations | 107 |
| 8.3 | Besov norm regularization and discretizations | 108 |
| 9 | Practical X-ray tomography with limited data | 111 |
| 9.1 | Sparse full-angle tomography | 114 |
| 9.2 | Limited-angle tomography | 119 |
| 9.3 | Low-dose three-dimensional dental X-ray imaging | 122 |
| 10 | Projects | 131 |
| 10.1 | Image deblurring | 132 |
| 10.2 | Inversion of the Laplace transform | 133 |
| 10.3 | Backward parabolic problem | 133 |
| II | Nonlinear Inverse Problems | 137 |
| 11 | Nonlinear inversion | 139 |
| 11.1 | Analysis of nonlinear ill-posedness | 140 |
| 11.2 | Nonlinear regularization | 143 |
| 11.3 | Computational inversion | 144 |
| 11.4 | Examples of nonlinear inverse problems | 145 |
| 12 | Electrical impedance tomography | 159 |
| 12.1 | Applications of EIT | 160 |
| 12.2 | Derivation from Maxwell's equations | 162 |
| 12.3 | Continuum model boundary measurements | 163 |
| 12.4 | Nonlinearity of EIT | 165 |
| 12.5 | Ill-posedness of EIT | 165 |
| 12.6 | Electrode models | 170 |
| 12.7 | Current patterns and distinguishability | 173 |
| 12.8 | Further reading | 180 |
| 13 | Simulation of noisy EIT data | 185 |
| 13.1 | Eigenvalue data for symmetric σ | 185 |
| 13.2 | Continuum model data and FEM | 187 |
| 13.3 | Complete electrode model and FEM | 191 |
| 13.4 | Adding noise to EIT data matrices | 197 |

| | | |
|-----------|---|------------|
| 14 | Complex geometrical optics solutions | 199 |
| 14.1 | Calderón's pioneering work | 199 |
| 14.2 | The $\bar{\partial}$ operator and its kin | 203 |
| 14.3 | CGO solutions for the Schrödinger equation | 205 |
| 14.4 | CGO solutions for the Beltrami equation | 215 |
| 15 | A regularized D-bar method for direct EIT | 223 |
| 15.1 | Reconstruction with infinite-precision data | 224 |
| 15.2 | Regularization via nonlinear low-pass filtering | 231 |
| 15.3 | Numerical solution of the boundary integral equation | 234 |
| 15.4 | Numerical solution of the D-bar equation | 237 |
| 15.5 | Regularized reconstructions | 242 |
| 16 | Other direct solution methods for EIT | 249 |
| 16.1 | D-bar methods with approximate scattering transforms | 249 |
| 16.2 | Calderón's method | 259 |
| 16.3 | The Astala–Päivärinta method | 266 |
| 16.4 | The enclosure method of Ikehata | 277 |
| 17 | Projects | 281 |
| 17.1 | Enclosure method for EIT | 281 |
| 17.2 | The D-bar method with Born approximation | 283 |
| 17.3 | Calderón's method | 285 |
| 17.4 | Inverse obstacle scattering | 286 |
| A | Banach spaces and Hilbert spaces | 291 |
| B | Mappings and compact operators | 293 |
| C | Fourier transform and Sobolev spaces | 297 |
| C.1 | Sobolev spaces on domains $\Omega \subset \mathbb{R}^n$ | 297 |
| C.2 | Fourier series and spaces $H^s(\partial\Omega)$ | 301 |
| C.3 | Traces of functions in $H^m(\Omega)$ | 306 |
| D | Iterative solution of linear equations | 307 |
| | Bibliography | 311 |
| | Index | 349 |