

Chapter 2

Complementary Material

2.1 Examples Using the Adaptive Control Toolbox

For the plants which can be described by a transfer function or an autoregressive moving average (ARMA) model, the signals z and ϕ of the corresponding linear parametric models (2.1) and (2.2) can be generated numerically using the Adaptive Control Toolbox. Given the transfer function or the ARMA model of the plant, the toolbox computes the parametric model signals z and ϕ dynamically, as illustrated by the following examples.

Example 2.1.1 Consider the plant

$$y = \frac{2s - 1.5}{s^2 + 3s + 2} u$$

and the corresponding SPM

$$z(t) = \theta^{*T} \phi(t),$$

where

$$\begin{aligned} z &= \frac{s^2}{\Lambda(s)} y, \\ \phi &= \left[\frac{s}{\Lambda(s)} u, \frac{1}{\Lambda(s)} u, -\frac{s}{\Lambda(s)} y, -\frac{1}{\Lambda(s)} y \right]^T, \\ \theta^* &= [b_1, b_0, a_1, a_0]^T = [2, -1.5, 3, 2]^T, \end{aligned}$$

and $\Lambda(s) = (s+1)^2$. Let $u(t) = \sin(\frac{\pi}{5}t)$. For zero initial conditions, the I/O history of the model is plotted in Figure 2C.1. These plots can be produced using the built-in MATLAB functions or by using the command `ufilt*` of the Adaptive Control Toolbox.

For online computation of the signals z and ϕ while the output y is being generated, we use the command `utf21m*`. The arguments needed to run the command are the orders of the plant numerator and denominator polynomials (in this example, $m=1$ and $n=2$, respectively), and the values of the input u and output y at each instant of time. The results using `utf21m` are shown in Figure 2C.2. The same results can be obtained using the Simulink block `Parametric Model*` as well.

* For further information about this command, refer to the toolbox manual—B. Fidan and P. A. Ioannou, *Adaptive Control Toolbox for Use with MATLAB and Simulink: User's Guide* (available from the authors), 2006.

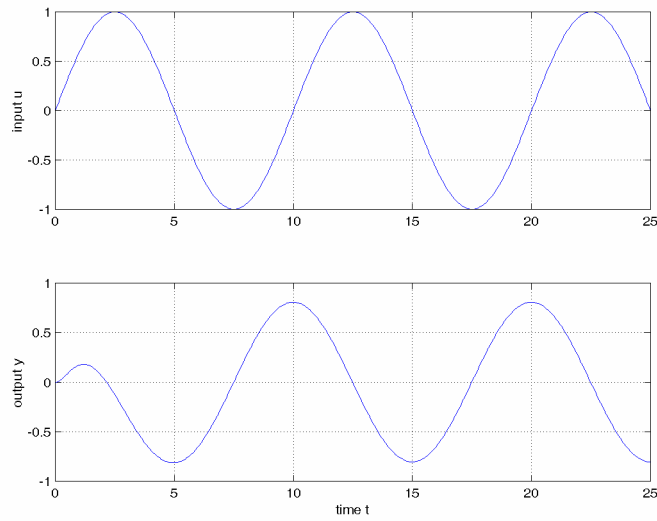


Figure 2C.1 I/O history of the plant in Example 2.1.1.

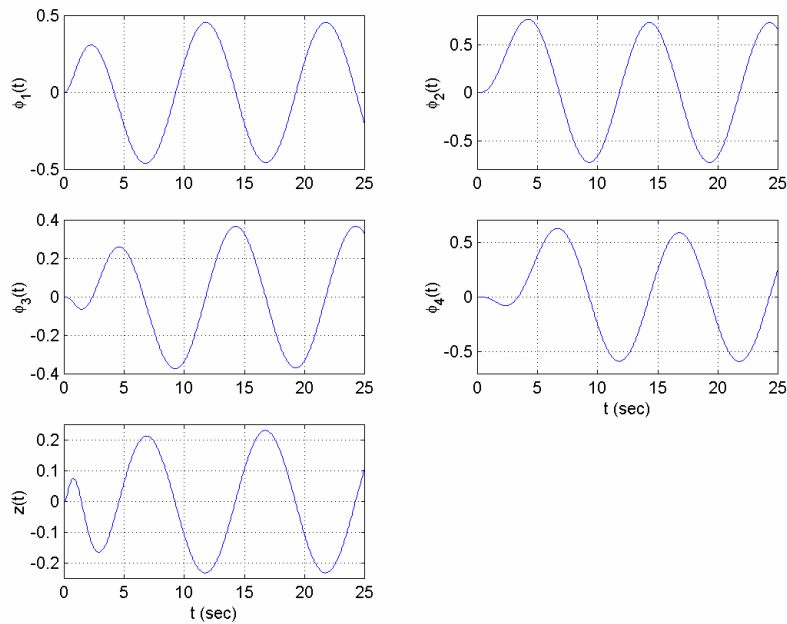


Figure 2C.2 Time history of the signals ϕ and z in Example 2.1.1.

Assume that parameters a_0 and b_1 are known and the unknown parameters are a_1 and b_0 . The SPM in this case is

$$z_r = \theta_r^{*T} \phi_r,$$

where

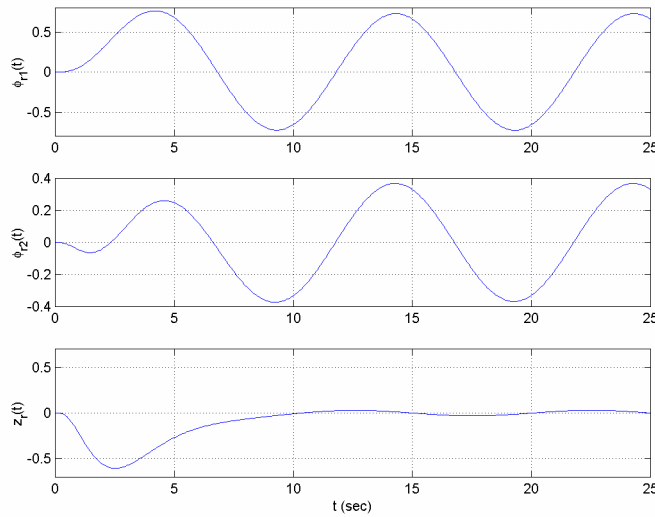


Figure 2C.3 Time history of the signals ϕ_r and z_r in Example 2.1.1.

$$z_r = \frac{s^2 + 2}{\Lambda(s)} y - \frac{2s}{\Lambda(s)} u,$$

$$\phi_r = \left[\frac{1}{\Lambda(s)} u, -\frac{s}{\Lambda(s)} y \right]^T,$$

$$\theta_r^* = [b_0, a]^T = [-1.5, 3]^T.$$

The function **lmred*** can be used to generate the signals of this lower-order SPM as shown below:

```
P = lmred('tf', [n m], 0, 2, 1, 2);
[zc, phir] = lmred(z, phi, P);
```

Here, the third and the fifth arguments of **lmred** are vectors of indices of the known denominator and numerator parameters, and the fourth and the sixth arguments are the corresponding parameter values. The time history of the parameters z_r and ϕ_r obtained is shown in Figure 2C.3. The same results can be obtained using the Simulink block **Model Reduction*** as well. ■

Example 2.1.2 Consider the ARMA model

$$y(k) = 1.9y(k-1) - 0.9y(k-2) + u(k-1) + 0.5u(k-2) + 0.25u(k-3),$$

which can be rewritten in the form of an SPM as follows:

$$z(k) = \theta^{*T} \phi(k),$$

where

* For further information about this command, refer to the toolbox manual.

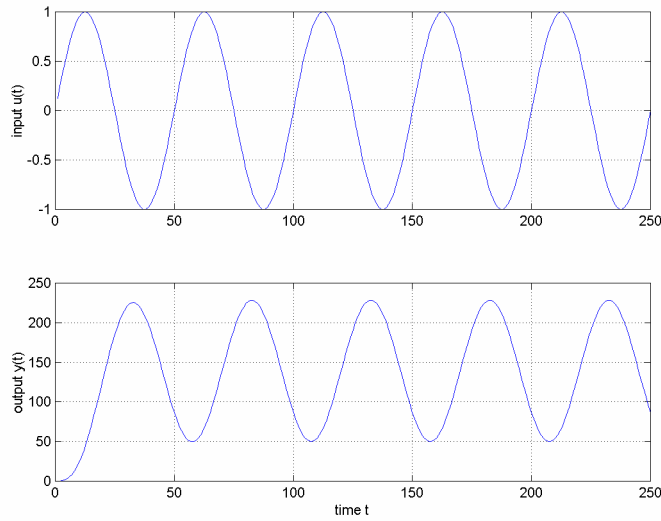


Figure 2C.4 I/O history of the ARMA model in Example 2.1.2.

$$z(k) = y(k),$$

$$\phi(k) = [u(k-1), u(k-2), u(k-3), -y(k-1), -y(k-2)]^T,$$

$$\theta^* = [b_0, b_1, b_2, a_1, a_2]^T = [1, 0.5, 0.25, -1.9, 0.9]^T.$$

Let $u(k) = \sin\left(\frac{\pi}{25}k\right)$. For zero initial conditions, the I/O history of the model, which can be obtained using `ufilt*`, is shown in Figure 2C.4. The time history of the corresponding SPM signals z and ϕ can be obtained offline using the toolbox command `io21m*`. Alternatively, one can use `uarma21m*` in order to perform the same task on-line. The results are exactly the same and are presented in Figure 2C.5. The same results can be obtained using the Simulink block `Parametric Model*` as well.

If the coefficients a_2 , b_0 , and b_1 are known, the SPM can be reduced to be in terms of only a_1 and b_2 . The reduced SPM is given by

$$z_r(k) = \theta_r^{*T} \phi_r(k),$$

where

$$z_r(k) = y(k) + y(k-2) - u(k-1) - 0.5u(k-2),$$

$$\phi_r(k) = [u(k-3) \quad -y(k-1)]^T,$$

$$\theta_r^* = [b_2 \quad a_1]^T = [0.25 \quad -2]^T.$$

* For further information about this command, refer to the toolbox manual.

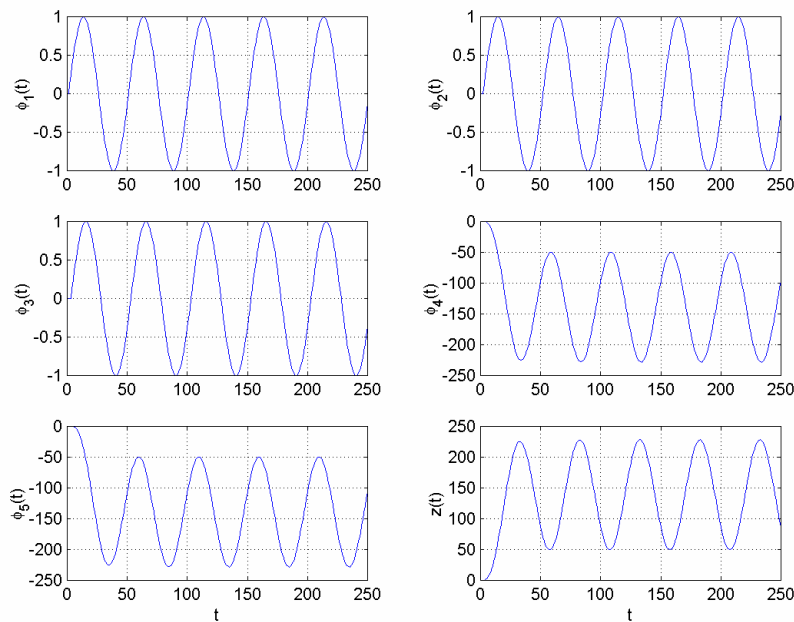


Figure 2C.5 Time history of the signals ϕ and z in Example 2.1.2.

The time history of the signals z_r and ϕ_r is obtained using **lmred**^{*} as follows:

```
P = lmred('arma', [n d m], 2, 0.9, [0 1], [1 0.5]);
[zr, phir] = lmred(z, phi, P);
```

The third and the fifth arguments of **lmred** are vectors of indices of known AR and MA parameters, and the fourth and the sixth arguments are the corresponding parameter values. The results are shown in Figure 2C.6. The same results can be obtained using the Simulink block **Model Reduction**^{*} as well. ■

^{*} For further information about this command, refer to the toolbox manual.

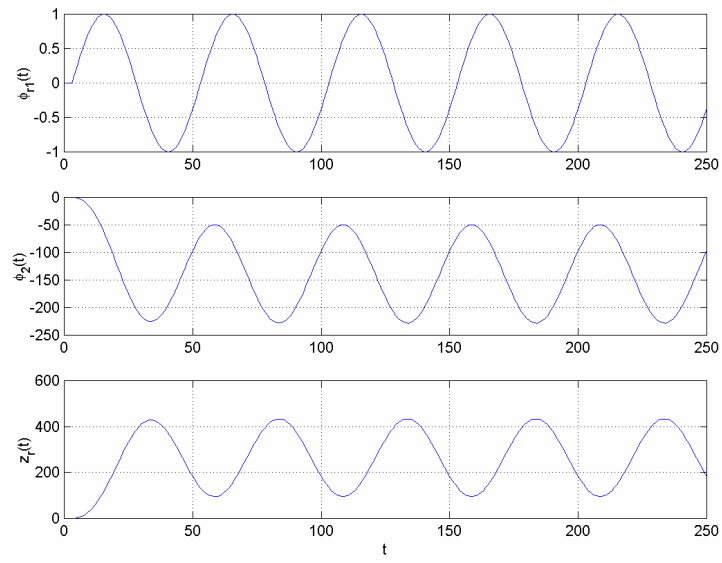


Figure 2C.6 Time history of the signals ϕ_r and z_r in Example 2.1.2.