

Preface

The interconnection between two (or more) physical systems is always accompanied by *transfer* phenomena (material, energy, information), such as *transport* and *propagation*. Mathematically speaking, transport and propagation phenomena can be represented by *delay elements*. In this way the corresponding overall systems are governed by a special type of differential equations, namely *delay differential equations*.

Delay differential equations are also used in modeling various other phenomena coming from biosciences (heredity in population dynamics [143, 167]), chemistry (behaviors in chemical kinetics [289, 259]) or economy (dynamics of business cycles [290]). Further examples in engineering can be found in [285, 222, 95].

As mentioned by Elsgolts and Norkin [70] or Răsvan [260], time-delay systems have a long history and, to the best of our knowledge, the first delay differential equations are encountered in the work of Bernoulli and Condorcet. However, the theory started to be developed in the second-half on the 20th century with the work of the East-European Mathematical school: Myshkis [218], Krasovskii [141] and Halanay [99] (to cite only a few), who devoted most attention to the extension of the Lyapunov theory to such class of differential equations. In the 60s, an increasing interest in the topic appeared also in North-America as confirmed by the monographs of Pinney [250] and Bellman and Cooke [13], the first one almost forgotten, with a particular interest in complex-domain approach and related frequency-domain techniques and methods. Next, the theory arrived to some degree of maturity in the 70s as proven by the publications and the monographs devoted to the field in that period. Among them, we mention the pioneering work of Hale [100] (the second edition of the monograph published in 1971), which is one of the most cited reference in the field for the fundamental results and approaches, but also for the quality and the clarity of the presentation. For further references and a deeper historical perspective, we refer to [222, 255, 234].

It is important to point out that various references devoted to time-delay systems in engineering exist even before the 50s as, for example, the papers co-authored by Calder [39, 40] and the editorial of the *Engineer* journal [284], with some contradictory conclusions concerning the effects induced by the delay presence in dynamical systems: sometimes *destabilizing* (mainly by using “huge” gains), and sometimes *stabilizing* (mainly in controlling some oscillatory modes). The explanation of such “dichotomic” behaviors was done case-by-case, without any attempt for a comprehensive explanation of the situations where stabilizing/destabilizing effects may occur.

Although by now the fundamental results in the theory of functional differential equations are well-known and well-understood (see, for instance, [13, 100, 105] to cite only a

few), the increasing number of applications involving large-scale systems with corresponding complex decision making strategies in which the *delay* (transport, propagation, communication, decision) becomes a “critical” parameter made the development of efficient numerical algorithms and methods for evaluating critical delays and related stability/instability properties necessary. This monograph presents some approaches and techniques in this sense.

Recent approaches in *robust control* opened interesting perspectives and issues in dealing with delays in dynamical systems, where delays are eventually treated as *uncertainty* [95, 222, 20]. Some of them (frequency-sweeping tests, matrix pencil approaches) will be largely discussed in the monograph. Such interpretations of delays as uncertainty are at the origin of an abundant literature in the control area. The corresponding results are expressed in terms of solutions of appropriate Riccati equations [169], linear matrix inequalities [20] in connection or not with the μ -formalism. An exhaustive overview concerning these approaches in the context of stability analysis can be found in [222].

At the same time the increasing number of efficient algorithms for dealing with *nonlinear eigenvalue problems* [178] represented another important issue in treating delay systems. As in the finite-dimensional case, essential properties of time-delay systems (asymptotic behavior, stability, instability, oscillations) are connected with the spectrum location of the corresponding linearized systems. As we shall explain in the next chapters, time-delay systems are infinite-dimensional systems, but with particular spectral properties. Such properties will be explicitly exploited in deriving the main (stability and stabilization) results and related algorithms. In this context, a particular attention will be paid to the distinction between *retarded* and *neutral* systems because, although both belong to the class of time-delay systems, the spectral properties are distinct. Most of the approaches presented in this book concern *retarded* delay systems, yet they can be easily extended to the *neutral* case.

It is important to point out that, excepting the functional differential equations based representation, there are several ways to *represent* time-delay systems - as evolution equations over abstract spaces [15] (infinite-dimensional setting), 2-D systems [157], systems over rings of operators [129], and behavioral based representations [89]. Throughout the volume, we adopted the functional differential equation based representation. We further assume that the nominal models are completely known. In other words, we do not focus on delay modeling, identification or identifiability.

Book outline and content

Our intention is to present the stability analysis and synthesis by delayed (state and output) feedback in the linear case by using a *unitary* methodology - the *eigenvalue-based approach*. Without any loss of generality we mainly concentrate on the following aspects that, to our best knowledge, have not received a full treatment in the literature:

- (a) Sensitivity analysis with respect to delays and to other systems' parameters (continuity of the spectrum with respect to the parameters based on Rouché type theorems and variants, pseudospectra and related properties);
- (b) Pole placement strategies in stabilization and (nonlinear) optimization of the spectral abscissa function or robustness indicators. Although such approaches are rather

classical in the finite-dimensional case, the extensions to delay systems need some special treatment due to the infinite-dimensional nature of the system. However, the particular spectral properties will be helpful to perform such control strategies in both the retarded and the neutral cases, with some precautions in the latter case;

A lot of examples complete the presentation, and illustrate the main results proposed in the monograph. Most of the major ideas are explained by using (several) extremely simple, “easy-to-follow” (low-order) examples. Finally, the last part of the monograph is devoted to several applications spanning various fields from engineering to biology. All the applications considered start from some generic remarks on the way in which the models are derived, but without any deep discussions on the model derivation and its limitations. The choice of the applications was mainly motivated by their impact in engineering, biosciences, and related fields, but also by our own interest in the corresponding topics.

How to read the book?

The book is organized in three parts:

- (a) *Stability and robust stability.* This part deals with the *analysis* of linear time-delay systems from a stability point of view. It starts with an overview of spectral properties of both retarded and neutral systems. To make the fundamental results apparent, eigenvalue plots are used extensively throughout the text. Then the robustness of stability and related problems are studied using pseudospectra and related quantities such as stability radii. The next three chapters deal with the characterization of stability regions in parameter spaces, both qualitatively (shape of regions etc.) and quantitatively (explicit computational algorithms). Finally, extensions of the presented results for systems with constant parameters to systems with periodically varying parameters are briefly discussed.
- (b) *Stabilization and robust stabilization.* The second part is devoted to the *synthesis* problems that correspond to the analysis problems treated in the first part, with the focus on stabilization. The first chapter is devoted to an eigenvalue based stabilization approach that is inspired by the classical pole placement method for systems without delay. Next, a numerical case study is presented to illustrate how delays in the control loop affect the stabilizability with state feedback. The following chapter is devoted to the robust stabilization problem, and corresponds to the dual of the chapter on pseudospectra presented in the first part. Finally, a new stabilization approach is presented which is based on recently developed methods for non-smooth optimization.
- (c) *Applications.* A wide class of applications is presented, from congestion analysis in high-performance networks to output feedback stabilization by using the delays as controller parameters predictor-type controllers, from consensus problems in traffic flows to the stability analysis of various delay models in biosciences. We tried to have the right correlation between the theory presented in the first two parts of the monograph and the applications which we consider. In some cases, we present several alternative approaches handling the same stability analysis or control problem.

It is important to point out that we have made the parts independent each-other as much as possible. However, a number of fundamental results are needed for the whole theoretical development, and are presented in the first chapter of the monograph. Since such results can be found in excellent references devoted to the theory of functional differential equations, we decided only to mention them here, and to pay more attention to some particular approaches and related methodologies that have not received a full attention in the literature, as for example, the *sensitivity analysis* (to cite only one approach).

Acknowledgments

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WIM MICHIELS

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SILVIU-IULIAN NICULESCU.