Preface

Everything should be as simple as it is,
but not simpler.
—Albert Einstein (1879-1955).

A mathematical theory is not to be considered complete
until you have made it so clear that you can explain it
to the first man whom you meet on the street.
—David Hilbert (1862-1943).

Always take a pragmatic view in applied mathematics:
the proof of the pudding is in the eating.
—N. H. Bingham and Rüdiger Kiesel (2004) [33].

Overview of This Book

The aim of this book is to be a self-contained, practical, entry level text on stochastic processes and control for jump-diffusions in continuous time, technically Markov processes in continuous time.

The book is intended to be accessible to graduate students as well as a research monograph to researchers in applied mathematics, computational science and engineering. In fact, a number of colleagues have said that they would like to learn about stochastic processes, but have found it difficult to learn about it from the existing literature. Also, the book may be useful for practicianers of financial engineering who need fast and efficient answers to stochastic financial problems. Hence, the exposition is based upon integrated basic principles of applied mathematics, applied probability and computational science. The target audience includes mathematical modelers and students in many areas of science and engineering seeking to construct models for scientific applications subject to uncertain environments. The prime interest is in modeling and problem solving. The utility of the exposition, based upon systematic derivations along with essential proofs in the spirit of classical applied mathematics, is more important to setting up a stochastic model of an application than abstract theory. However, a lengthy last chapter is intended to bridge the gap between the applied world and the abstract world in order to enable applied students and readers to understand the more abstract literature.

More rigorous theorem formulation and proving is not of immediate importance compared to modeling and solving an applied problem, although many proofs are given here. Many research problems deal with new applications and often these new applications require models beyond those in the existing literature. So, it is important to have a reasonably understandable derivation for a nearby model that can be perturbed to obtain a proper new model. The level of rigor here is embodied in correct and systematic derivations, with many proofs and results not available elsewhere, under reasonable conditions, not necessarily the tightest possible conditions. In fact, much of this book and the theory of Markov processes in continuous time is based upon modifying the formulations for continuous functions in regular and advanced calculus to extend them to the discontinuous and non-smooth functions of stochastic calculus.

Origin of the Book

The book is based upon the author's courses *Math 574 Applied Optimal Control*, *Math 590 Special Topics: Applied Stochastic Control*, *MCS 507 Mathematical, Statistical and Scientific Software for Industry* and partly on *MCS 571 Numerical Methods for Partial Differential Equations*. In addition, the results from research papers on computational stochastic dynamic programming, computational finance and computational biomedicine are included. Courses in asymptotic analysis and numerical analysis play a role as well. However, as with lectures, every attempt is made to keep the book self-contained through an integrated approach, without depending heavily on prerequisites, especially with a diverse readership and interdisciplinary topics.

This book integrates many of the research and exposition advances made in computational stochastic dynamic programming and stochastic modeling. They exhibit the broader impact of the applications and the computationally oriented approach. The stochastic applications are wide-ranging, including the optimal economics of biological populations in uncertain and disastrous environments, biomedical applications in cancer modeling and optimal treatment, and financial engineering with applications in option pricing and optimal portfolios.

How This Book is Organized and How to Use It

- The prerequisites are too numerous to expect of any reader of a wide-ranging interdisciplinary book such as this one, so a single online Appendix B on the essentials of probability theory, mathematical analysis, matrix algebra and other topics can be found at

  `http://www.math.uic.edu/~hanson/math574/#Text`.

  Over-specification of prerequisites tend to filter out too many students who could benefit from this material. This appendix is intended to bring all readers up to the same level by self-study, where necessary, of the basic concepts and notations of probability and analysis needed for jump-diffusion processes and their deviations from continuity. It is not meant to be taught or read in sequence, but to include relevant results when needed and to make the presentation as self-contained as possible. Originally, this material as a pre-appended chapter, but became an appendix to reduce the size and cost of this book.

- Simple jump-diffusion Chapters 1, 2, 3 and 4 cover the basics for simple jump-diffusions, i.e., stochastic diffusion (Wiener of Brownian motion) and simple Poisson driven processes, including stochastic integration based upon Itô's computationally motivated mean square convergence for Markov processes and stochastic calculus for transformations of stochastic differential equations (SDEs). The speed and depth of coverage for the student or reader will depend on their level of knowledge, particularly with respect to prior knowledge of probability and diffusion processes which are more well known. The presentation is more elementary than that of later chapters to reduce the likelihood that readers will get lost at the basic level.

- Advanced and special topics are found in Chapters 5 to 12 and can be selected according to the instructor's or reader's interests. There are more chapters than can be covered in any one course.

  - Chapter 5 *Stochastic Calculus for General Markov SDEs* covers more advanced and general topics for SDEs. These include jumps driven by compound Poisson or Poisson random measure processes that allow randomly distributed jump-amplitudes, state-dependent jump-diffusions and multidimensional jump-diffusions.

  - Chapter 6 *Stochastic Optimal Control - Stochastic Dynamic Programming* and Chapter 8 *Computational Stochastic Control Methods* can form a stochastic control theory and computational components of a course. Also, if a chapter on deterministic control theory is desired for introduction and contrast to stochastic control, another online Appendix A *Deterministic Optimal Control* is available at the above listed URL. Appendix A gives a summary of deterministic optimal control results to provide a background for comparison to the stochastic optimal control results, but could be skipped if a deterministic control course is a prerequisite or if only stochastic optimal control are of interest. In Chapter 6 stochastic optimal control problems are introduced and the equation of stochastic dynamic programming is systematically derived from the basic principles of applied mathematics. Chapter 8 has treatments using either modified finite difference methods for optimal control problem or the Markov chain approximation methods. Computational methods are important for stochastic optimal control problems because there are so few exact analytical solutions. Computational methods are important for stochastic optimal control problems because there are so few exact analytical solutions.

- Chapter 7 *Kolmogorov Equations* concerns partial differential equation (PDE) methods for solving stochastic problems using the forward and backward Kolmogorov equations, Dynkin's integral formulas (also Feynmann-Kac's as Dynkin's with an integrating factor) that help provide PDE solutions without directly solving the PDE, boundary conditions and stopping time problems. Knowledge of partial derivatives from advanced courses in calculus is all that should be needed, a course in PDEs will be of little help, since a course is not essential and only these integral formulas are used in this chapter. PDE methods are an applied alternate to the abstract method of using martingales to solve stochastic problems, such as those in finance (see Chapter 12 for martingale and other abstract approaches.)

- Chapter 9 *Stochastic Simulations* contains treatments for direct simulations of SDEs and general simulations by the Monte Carlo method. This chapter along with Chapter 8 on computational dynamic programming could form a computational component of a course.

- Chapter 10 on financial applications and Chapter 11 on biomedical applications provide substantial examples of application of the theory and techniques treated in this book. Chapter 10 explains Merton's mathematical justification and generalization of the classical Black-Scholes option pricing problem in sufficient detail for those familiar with the diffusion processes properties in Chapters 1-4 and is a good motivating application for Chapter 5. Also treated are option pricing models for jump-diffusions, optimal portfolio and consumption models, and an important events model that modifies the jump-diffusion model with a quasi-deterministic jump model for scheduled announcements and random responses. Chapter 11 includes applications to stochastic optimal control or bioeconomic models, diffusion approximation models of tumor growth and a deterministic optimal control model of PDE-driven drug delivery model for the brain.

- Chapter 12 is an applied description of abstract probability methods, including probability measure, probability space, martingales and change in probability measure using either Radyn-Nikodým and Girsanov theorems. The last section is a generalization of jump-diffusions called Lévy processes that permit the jump-rate to be infinite. This chapter is meant to be a bridge between the applied view of stochastic processes and the abstract view to ease the transition to reading some of the more abstract literature on stochastic processes. However, depending on the instructor or reader, parts of this chapter can be woven into the coverage of the earlier chapters. For instance, a colleague said that Girsanov's measure change transformation was needed in his financial applications course and there are a pure diffusion version and a jump-diffusion version of the Girsanov theorem in this chapter.

Distinct Features of This Book

The book is based upon a number of distinct features:

- Both analytical and computational methods are emphasized based on the utility, with respect to the computational complexity, of the problems. Exercises and examples in the elementary chapters include both computational and analytic ones. Students need to have good analytic and computational skills to do well, since diverse skills are needed for many jobs.

- The treatment of jump and diffusion processes is balanced as well, rather than a stronger or nearly exclusive emphasis on diffusion processes. This is a unique feature of this book. This treatment of jump-diffusions is important for training graduate students to do research on stochastic processes, since the analysis of diffusion processes is so well-developed, there are many opportunities for open problems on jump-diffusions.

- It clearly shows the strong role that discontinuous as well as non-smooth properties of stochastic processes play compared to the random properties by emphasizing a concrete jump calculus, without much reliance on measure-theoretic constructs, except for the very useful random Poisson measure concept.

- Basic principles of probability theory in the spirit of classical applied mathematics are used to set up the practical foundations through clear and systematic derivations, making the book accessible as a research monograph to many who work with applications.

- It shows how analytical-canonical control problem models, such as the linear-quadratic, jump-diffusion (LQJD) problem and financial risk-adverse power utilities, can be used to reduce computational dimensional complexity of approximate solutions along with other computational techniques.

- Insightful and useful material are used so that the book can be readily used to model realistic applications and even modify the derivations when new applications do not quite fit the old stochastic model.

- Clear explanations for the entry level student are used. In particular, clear and consistent notation is used, such that the notation is clearly identified with the quantity it symbolizes, rather than arbitrarily selected. Sometimes this has meant some compromise on some standard notation, for instance, P is used for the Poisson process to be consistent with the W used for the Wiener process. This means that P could not be used for probability, so Prob is used in place of P (or Pr) and is clearer to a diverse audience. Similarly, probability distributions are denoted by by Φ and densities by ϕ since P is used for Poisson and F is used for transformation functions throughout the book.

Target Audience

Colleagues and students have requested a more accessible, practical treatment of these topics. They are interested in learning about stochastic calculus and optimal stochastic control in continuous time, but reluctant to invest time to learn it from more advanced treatments relying heavily on abstract concepts. Hence, this book should be of interest to an interdisciplinary audience of applied mathematicians, applied probabilists, engineers (including control engineers dealing with deterministic problems and financial engineers needing fast as well as useful methods for modeling rapidly changing market developments), statisticians and other scientists. After this primary audience, a secondary audience would be mathematicians, engineers and scientists, using this book as a research monograph, seeking more intuition to more fully understand stochastic processes and how the more advanced analytical approaches fit in with important applications like financial market modeling.

Prerequisites

For optimal use of this book, it would be helpful to have had prior introduction to applied probability theory including continuous random variables, mathematical analysis at least at the level of advanced calculus. Ordinary differential equations, partial differential equations and basic computational methods would be helpful but the book does not rely on prior knowledge of these topics by using basic calculus style motivations. In other words, the more or less usual preparation for students of applied mathematics, science and engineering should be sufficient. However, the author has strived to make this book as self-contained as practical, not strongly relying on prior knowledge and explaining or reviewing the prerequisite knowledge at the point it is needed to justify a step in the systematic derivation of some mathematical result. Online Appendix B supplies essential preliminaries.

MATLAB™ Computation

As part of the theme of balancing computation and analysis, MATLAB™, the matrix laboratory computation system is used for almost all computational examples and figure illustrations. Simple MATLAB™ codes are described in class and the code for all text figures are given in online Appendix C at the URL previously cited in both listing and source forms. MATLAB™ greatly facilitates the development of code and is ideally suited to stochastic processes and control problems. Also, MATLAB™ now comes with the Maple™ kernel built into the MATLAB™ student package for including elementary symbolic computations with numeric computations. Beyond the initial elementary assignments, the students are required to submit their assignments with professionally done illustrations for which they can find examples in online Appendix C. Many students surveyed at the end of the class actually list MATLAB™ with the other topics that they were happy to learn. MATLAB™ is also helpful later for producing professional research papers and theses.

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Applebaum [12], Arnold [13], Bingham and Kiesel [33], Bliss [40], Çinlar [56], Clark [57], Cont and Tankov [60], Feller [84, 85], Fleming and Rishel [86], Gihman and Skorohod [95, 96], Goel and Richter-Dyn [99], Glasserman [97], Hammersley and Handscomb [105], D. Higham [140, 141], Hull [148], Itô [150], Jäckel [151] Jazwinski [155], Karlin and Taylor [162, 163, 265] Kirk [164], Kloeden and Platen [166], Kushner [174, 176], Kushner and Dupuis [179], Ludwig [187], Merton [203], Mikosch [209], Øksendal [222], Øksendal and Sulem, [223], Parzen [224], Protter [232], Runggaldier [239], Schuss [244], Snyder and Miller [252], Tuckwell [270], Steele [256], Wonham [286], and others. Although this influence may not be directly apparent here, some have shown how to make the presentation much simpler, while others have supplied the motivation to simplify the presentation, making it more accessible to a more general audience and other applications.

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Many of our national supercomputing centers have provided supercomputing time on the the currently most powerful supercomputers for continuing research for solving large scale stochastic control problems in *Advanced Computational Stochastic Dynamic Programming* and also for computational science education. In addition to Argonne National Laboratory, these were National Center for Supercomputing Applications (NCSA), Los Alamos National Laboratory's Advanced Computing Laboratory (LANL/ACL), Cornell Theory Center (CTC/CNSF), Pittsburgh Supercomputing Center (PSC) and the San Diego Supercomputing Center (SDSC/NPACI) during 1987-2003.

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