

Contents

Preface	ix
1 Introduction	1
1.1 Boundary Control	1
1.2 Backstepping	2
1.3 A Short List of Existing Books on Control of PDEs	2
1.4 No Model Reduction in This Book	3
1.5 Control Objectives for PDE Systems	3
1.6 Classes of PDEs and Benchmark PDEs Dealt with in This Book	3
1.7 Choices of Boundary Controls	4
1.8 The Domain Dimension: 1D, 2D, and 3D Problems	5
1.9 Observers	6
1.10 Adaptive Control of PDEs	6
1.11 Nonlinear PDEs	6
1.12 Organization of the Book	6
1.13 Why We Don't State Theorems	8
1.14 Focus on Unstable PDEs and Feedback Design Difficulties	9
1.15 The Main Idea of Backstepping Control	9
1.16 Emphasis on Problems in One Dimension	11
1.17 Unique to This Book: Elements of Adaptive and Nonlinear Designs for PDEs	11
1.18 How to Teach from This Book	11
2 Lyapunov Stability	13
2.1 A Basic PDE Model	14
2.2 Lyapunov Analysis for a Heat Equation in Terms of " L_2 Energy"	16
2.3 Pointwise-in-Space Boundedness and Stability in Higher Norms	19
2.4 Notes and References	22
Exercises	22
3 Exact Solutions to PDEs	23
3.1 Separation of Variables	23
3.2 Notes and References	27
Exercises	27

4	Parabolic PDEs: Reaction-Advection-Diffusion and Other Equations	29
4.1	Backstepping: The Main Idea	30
4.2	Gain Kernel PDE	31
4.3	Converting the Gain Kernel PDE into an Integral Equation	33
4.4	Method of Successive Approximations	34
4.5	Inverse Transformation	35
4.6	Neumann Actuation	41
4.7	Reaction-Advection-Diffusion Equation	42
4.8	Reaction-Advection-Diffusion Systems with Spatially Varying Coefficients	44
4.9	Other Spatially Causal Plants	46
4.10	Comparison with ODE Backstepping	47
4.11	Notes and References	50
	Exercises	50
5	Observer Design	53
5.1	Observer Design for PDEs	53
5.2	Output Feedback	56
5.3	Observer Design for Collocated Sensor and Actuator	57
5.4	Compensator Transfer Function	60
5.5	Notes and References	63
	Exercises	63
6	Complex-Valued PDEs: Schrödinger and Ginzburg–Landau Equations	65
6.1	Schrödinger Equation	65
6.2	Ginzburg–Landau Equation	67
6.3	Notes and References	75
	Exercises	76
7	Hyperbolic PDEs: Wave Equations	79
7.1	Classical Boundary Damping/Passive Absorber Control	80
7.2	Backstepping Design: A String with One Free End and Actuation on the Other End	83
7.3	Wave Equation with Kelvin–Voigt Damping	85
7.4	Notes and References	87
	Exercises	88
8	Beam Equations	89
8.1	Shear Beam	91
8.2	Euler–Bernoulli Beam	95
8.3	Notes and References	101
	Exercises	105
9	First-Order Hyperbolic PDEs and Delay Equations	109
9.1	First-Order Hyperbolic PDEs	109
9.2	ODE Systems with Actuator Delay	111
9.3	Notes and References	113
	Exercises	114

10	Kuramoto–Sivashinsky, Korteweg–de Vries, and Other “Exotic” Equations	115
10.1	Kuramoto–Sivashinsky Equation	116
10.2	Korteweg–de Vries Equation	117
10.3	Notes and References	118
Exercises	118
11	Navier–Stokes Equations	119
11.1	Channel Flow PDEs and Their Linearization	119
11.2	From Physical Space to Wavenumber Space	121
11.3	Control Design for Orr–Sommerfeld and Squire Subsystems	122
11.4	Notes and References	127
Exercises	128
12	Motion Planning for PDEs	131
12.1	Trajectory Generation	132
12.2	Trajectory Tracking	139
12.3	Notes and References	141
Exercises	141
13	Adaptive Control for PDEs	145
13.1	State-Feedback Design with Passive Identifier	146
13.2	Output-Feedback Design with Swapping Identifier	151
13.3	Notes and References	157
Exercises	158
14	Towards Nonlinear PDEs	161
14.1	The Nonlinear Optimal Control Alternative	162
14.2	Feedback Linearization for a Nonlinear PDE: Transformation in Two Stages	163
14.3	PDEs for the Kernels of the Spatial Volterra Series in the Nonlinear Feedback Operator	165
14.4	Numerical Results	167
14.5	What Class of Nonlinear PDEs Can This Approach Be Applied to in General?	167
14.6	Notes and References	170
Exercise	171
Appendix	Bessel Functions	173
A.1	Bessel Function J_n	173
A.2	Modified Bessel Function I_n	174
Bibliography		177
Index		191