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Practical Methods for Optimal Control and Estimation Using Nonlinear Programming. Second Edition. By John T. Betts. SIAM, Philadelphia, 2010. \$89.00. xiv+434 pp., hardcover. *Advances in Design and Control*. Vol. 19. ISBN 978-0-898716-88-7.

The first edition of this book was published in 2001 [1]. Two natural questions arise. If you have the first edition, is it worthwhile looking at the second edition? Second, does this reviewer have anything to say that was not said in his earlier reviews of the first edition such as [2]? The answer to the first question is yes, and hopefully the same is true for the second.

This book still occupies a unique place in the optimal control literature. The author has been active in the numerical control and optimization communities for over two decades. He is also the lead developer of the general purpose optimal control software SOCS (Sparse Optimal Control Software), which has been continuously refined and improved for over a decade at the Boeing Corporation. This software was designed to solve a very wide range of optimal control problems. Unlike in an academic environment, where one can choose one's application, in this industrial setting one cannot choose which problems to solve. One has to find a way to approach each problem presented, if at all possible, no matter how difficult. This book then represents the synthesis of experience in solving literally hundreds of challenging optimal control problems and developing a robust piece of software. Many of the recent enhancements are based on research that the author has carried out since the first edition.

To take an analogy from academics, the first edition is like a one-semester graduate course. The key ideas are presented. While not directly about SOCS, it presents a synthesis of control theory and the types of algorithms that serve as the basis for SOCS and some of the other software packages in existence. The second edition includes, almost intact, the first edition, but it also includes a large amount of additional information, enough to teach the second semester and a special topics course after-

wards! There are six types of additional material. First, there is mathematical material introducing interior point methods including barrier algorithms. Second, there are many more examples illustrating different computational issues. Third, there is a much more in-depth discussion of computational issues and choices. Fourth, this requires some more detail on implementation approaches such as differencing and splines. Fifth, there is now a major discussion of the very important problem of parameter estimation. Finally, there are some large scale complex examples that show how all the previous issues and ideas come together to solve real-world problems. Surprisingly, in spite of doubling in size and addressing much more technical detail in the later sections, the book maintains the clear and readable style of the first edition.

In order to understand why this book includes the material that it does, it is helpful to comment briefly on the approach adopted in SOCS, which is for the software to discretize everything including the dynamics and constraints. This is referred to as direct transcription. The advantage of this type of approach is that it is easy to incorporate all kinds of inequality and equality constraints. There are very few restrictions on the types of optimal control problems that can be formulated. Also, the only requirement, in principle, of the users is that they be able to write the problem in the syntax requested by SOCS. The resulting large discrete nonlinear programming program is then passed to an NLP solver.

Of course, as with all nonlinear optimal control software, there remains a number of problems such as conditioning, finding initial feasible iterates, etc.

Aerospace optimal control applications cover everything from spacecraft trajectories to manufacturing processes. These applications typically are highly nonlinear and have many and sometimes complex inequality constraints. By going to a direct transcription approach, the need to derive the optimality necessary conditions is avoided. In fact, for many of these problems it would be difficult for anyone, even experts in optimal control, to easily find useful forms of

the necessary conditions. As illustrated by examples and recent results, it is sometimes not even possible to find a useful solution for the necessary conditions.

There are a couple of other underlying aspects of the philosophy of this book which are worth commenting on since they may not be obvious to the casual reader. Since there is no attempt to form the necessary conditions of the original optimal control problem, optimality is not directly checked. The emphasis is on finding a useful answer. Thus the temporal discretization refinement process focuses on accurately resolving the dynamics. So, if you use the computed control, the dynamics will have been correctly resolved and the cost will be as computed. A close-to-optimal control is often good enough, particularly since robustness and other practical issues may rule out using the actual optimal control.

While direct transcription has many advantages in making it easier to set up problems and get a solution process underway, especially for nonexpert users, it has the added cost that the resulting NLP problems are quite large. This means that one cannot treat the discretization process and the optimization process differently. Sparse matrix techniques have to be used by the NLP solver, and these techniques have to be tuned to the specific discretizations used. The choice of discretization is made with the need for this tuning in mind. Thus the discretization and the sparse linear algebra and the NLP solver have to all be developed together. This can be a major undertaking, but, as illustrated by some of the computational examples in this book, the result can sometimes be the solution of a complex problem in surprisingly little time.

Several of my Ph.D. students have read this book. It does an excellent job of giving a readable overview of how a modern direct transcription code works, some of the mathematics that lies behind its algorithms, and many of the numerical issues that arise when solving problems. The presentation is accessible to students who have completed a first graduate level numerical analysis course. This book is recommended reading for any person who will be working with numerical optimal control. More

advanced readers, who are either designing or working with optimal control codes, while they can skip some of the introduction, will find the various comments and observations informative, thought provoking, and undoubtedly in some cases controversial. There are many successful approaches to solving optimal control problems; this book presents one widely used and often successful approach.

Finally, it should be pointed out that the choice of using direct transcription rather than a method such as control parameterization, which reduces the problem dimension by integrating the dynamics, is not just a question of allowing for more general problems at the price of increased NLP problem size. Recent research, to which author Betts was a major contributor, has suggested that there are some fundamental differences in numerical optimal control theory when direct transcription is used. For example, the Hermite–Simpson discretizations, although they are not convergent when used as a differential equation integrator when certain types of state constraints are active, still can be used successfully in a direct transcription code when those same constraints are active. Thus the theory of direct transcription codes is not a simple intersection of the theory for NLP problems and the theory for discretizations of ordinary differential equations but requires its own theory. For researchers who wish to get an overview of this approach, to find what some of the computational and theoretical issues are, and to discover how a code solves these problems, this book provides an excellent introduction and overview.

REFERENCES

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- [2] S. L. CAMPBELL, *Review of “Practical Methods for Optimal Control Using Nonlinear Programming” by John T. Betts*, *Internat. J. Robust Nonlinear Control*, 14 (2004), pp. 1019–1021.

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