Preface

The book is devoted to the theory and application of second-order necessary and sufficient optimality conditions in the Calculus of Variations and Optimal Control. The theory is developed for control problems with ordinary differential equations subject to boundary conditions of equality and inequality type and mixed control-state constraints of equality type. The book exhibits two distinctive features: (a) necessary and sufficient conditions are given in the form of no-gap conditions, and (b) the theory covers broken extremals, where the control has finitely many points of discontinuity. Sufficient conditions for regular controls that satisfy the strict Legendre condition can be checked either via the classical Jacobi condition or through the existence of solutions to an associated Riccati equation.

Particular emphasis is given to the study of bang-bang control problems. Bang-bang controls induce an optimization problem with respect to the switching times of the control. It is shown that the classical second-order sufficient condition for the Induced Optimization Problem (IOP), together with the so-called strict bang-bang property, ensures second-order sufficient conditions (SSC) for the bang-bang control problem. Numerical examples in different areas of application illustrate the verification of SSC for both regular controls and bang-bang controls.

SSC are crucial for exploring the sensitivity analysis of parametric optimal control problems. It is well known in the literature that for regular controls satisfying the strict Legendre condition, SSC allow us to prove the parametric solution differentiability of optimal solutions and to compute parametric sensitivity derivatives. This property has lead to efficient real-time control techniques. Recently, similar results have been obtained for bang-bang controls via SSC for the IOP. Though the discussion of sensitivity analysis and the ensuing real-time control technques are an immediate consequence of the material presented in this book, a systematic treatment of these issues is beyond the scope of this book.

The results of Sections 1.1–1.3 are due to Levitin, Milyutin, and Osmolovskii. The results of Section 6.8 were obtained by Milyutin and Osmolovskii. The results of Sections 2.1–3.4, 5.1, 5.2, 6.1, 6.2, and 6.5 were obtained by Osmolovskii; some important ideas used in these sections are due to Milyutin. The results of Sections 4.1 and 4.2 (except for Section 4.1.5) are due to Lempio and Osmolovskii. The results of Sections 5.3, 6.3, 6.6, and 7.1–7.5 were obtained by Maurer and Osmolovskii. All numerical examples in Sections 4.1, 5.4, 6.4, and Chapter 8 were collected and investigated by Maurer, who is grateful for the numerical assistance provided by Christof Büskens, Laurenz Göllmann, Jang-Ho Robert Kim, and Georg Vossen. Together we solved a lot more bang-bang and singular control problems than could be included in this book. H. Maurer is indebted to Yalçin Kaya for drawing his attention to the arc-parametrization method presented in Section 8.1.2.

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