Preface

Consider the time periods required

- for a driver to react in response to large disturbances in road traffic;
- for a signal sent from Houston to reach a satellite in orbit;
- for a coolant to be distributed among all air-conditioners in a residential building;
- for a job to be executed on a server;
- for a cell to reach a certain maturation level;
- for a relativistic particle to feel the electromagnetic force from another particle;
- for a cutting tool to perform two succeeding cuts.

The common attribute of all these time periods is that they do not remain constant.

Another feature in common to the dynamics of traffic flow, cooling systems, networks and queuing systems, population growth, and cutting processes is that they are all nonlinear. Although a plethora of techniques exist for the control of nonlinear systems without delays, control design for nonlinear systems in the presence of *long* delays with *large and rapid variation* in the actuation or sensing path, or delays affecting the internal states of a system, introduces significant feedback design challenges that have, heretofore, remained largely untackled.

In this book we present systematic design techniques applicable to general nonlinear systems with long, nonconstant delays. While there is a nearly inexhaustible number of combinations in which one or multiple delays (as well as discrete or distributed delays) can enter a dynamical system, we focus our attention on problems with input delays. Arguably, if the system has only a single discrete delay, the case where the delay affects the input (rather than some of the state components that appear on the system model's right-hand side) is the most challenging case for control, and in particular for stabilization. Hence, our focus on input delay problems is without much loss of generality.

In ODE systems with input delays, the overall state of the dynamical system consists of the vector state of the ODE and the functional state of the input delay. (If the delay is nonconstant, the support of the functional part of the state is nonconstant as well.) For a problem with such a (relatively) "unusual" state, the control design can be approached—in principle—in an abstract setting where the particular structure of the system and of the state is deemphasized and the design is performed in an abstract infinite-dimensional setting. However, at present, methods that fit such an abstract approach exist only when the ODE plant is linear (and when the delay is constant), but not when the plant is nonlinear.

To develop designs that are applicable to both linear and nonlinear plants, a much more structure-specific approach is needed. This approach exploits the structure of the system with input delay. In this approach the delay is "compensated for." On the surface, the approach appears very simple: the idea is to give up on controlling the current state over the immediate near term—because systems with input delays are not "small-time" controllable but only controllable over time intervals longer than the delay—and to, instead, design the feedback law to control the future state. To control the future state, the feedback law requires the value of the future state, rather than merely of the current state.

Knowledge of the future state only seems like an impossible thing to ask. It is not. The value of the state in the future can be expressed, using the system model, in terms of the current state and of the past inputs. Such a future state value is called a prediction and the formula for the state is called a "predictor." A feedback law employing the predictor—a formula for the future state—achieves successful control of the ODE system in the future, after an initial period of time equal to the input delay. Hence, in this approach one only needs to design a feedback law for the delay-free system and to construct the predictor.

The predictor construction for linear plants is straightforward (thanks to the variation of constants formula, using the current state as the initial condition). In the nonlinear case the predictor is not given explicitly, but the approach is conceptually the same, employing a predictor that depends on the current state and past inputs.

The *predictor-based* approach outlined above applies not only to systems (linear and nonlinear) with constant delays but also to systems with time-varying delays. It even applies to systems whose delays are time varying as a result of being dependent on the system state.

This book guides the reader from the basic idea of predictor feedback for linear systems with constant delays only on the input all the way through to nonlinear systems with state-dependent delays on the input as well as on system states.

What Does the Book Cover? While the most useful part of the book is the design of the feedback laws for systems with input delays and certain system structures involving state delays, the design is the easy part.

The key challenge is in the analysis of stability. While the ODE state is trivially endowed with stability-like properties after the initial time equal to the delay, the analysis of stability requires not only the quantification of the ODE state over this initial period, but also the quantification of the infinite-dimensional delay state over the entire time period, from zero to infinity.

To conduct such an analysis, we employ the recently introduced techniques based on infinite-dimensional backstepping transformations. These transformations employ linear or nonlinear Volterra operators of the delayed input state. In addition, the transformations involve the ODE state. The stability analysis of the predictor-based feedback laws employs Lyapunov functionals that incorporate the backstepping transformations, the inverses of the backstepping transformations, and the complex nonlinear relationships between the Lyapunov functionals and the norms of the overall system state (combining the vector state and the functional state).

Although the book's emphasis is on heretofore intractable problems involving nonlinear systems with time-varying and state-dependent delays, we also provide designs of predictor feedback laws for linear systems with constant distributed delays and known or unknown plant parameters, and for linear systems with simultaneous known or unknown constant delays on the input and the state.

Our results are always accompanied by a stability analysis which we perform by constructing Lyapunov–Krasovskii functionals for each particular problem. With our Lyapunov functionals we provide, explicitly, performance measures of the closed-loop system such as convergence rate and overshoot. In addition, the Lyapunov functionals that we construct allow us to quantify the robustness properties of our control laws to plant uncertainties (including delays), as well as to exogenous disturbances.

This book's most advanced results are the ones for state-dependent delays. State-dependent delays introduce a puzzling challenge. For input delays that are time varying—whether as a result of a direct dependence of the delay on time or of an indirect dependence of the delay on time thanks to the delay's dependence on the system's state—the challenge in constructing the predictor is that the time horizon over which prediction should be conducted is not in general equal to the length of the (time-varying) delay. This is illustrated in the cover art for our book. The prediction horizon depends on an inverse function of the function—which we refer to as the "delayed time"—which is given as a difference between the current time and the current delay. When the delay is state dependent, the "delayed time" function is not known a priori in the future. As a result, the "delayed time's" inverse function, which determines the prediction horizon, is not known at present time. In fact, the prediction horizon depends on the future predictor state. This gives rise to a seemingly intractable, seemingly circuitous situation, in which the predictor state is calculated over a time period that depends on the predictor state itself. We resolve this quandary and give a design formula for the predictor state even for state-dependent delays. We also provide a stability analysis, in which we overcome challenges associated with the fact that the support of the functional state (the input delay state) depends on the value of the vector state of the ODE plant.

Who Is This Book For? This book should be of interest to researchers working on control of delay systems, including engineers, mathematicians, and students. They may find many parts of it quite fascinating since it provides elegant and systematic treatments of long-standing problems that arise in many applications.

First and foremost among research communities that may benefit from the book, mathematicians working on nonlinear ordinary and functional differential equations, as well as on partial differential equations, may be stimulated by the wealth of mathematical challenges that arise in the systems considered in this book, particularly by systems with simultaneous input and state delays and systems with state-dependent delays.

All of our designs are given by explicit formulae. Therefore, the book should be of interest to any engineer who has faced delay-related challenges and is concerned with actual implementations: electrical and computer engineers who encounter varying delays imposed by communication networks; mechanical and other manufacturing engineers that are forced to operate their machinery within conservative bounds, since otherwise the uncompensated delay can lead to instability; and aerospace engineers working on combustion engines. Even civil engineers come across challenges due to the presence of long, varying delays in terms of traffic flow dynamics, or water and gas distribution dynamics. All of them may find this book useful because it provides systematic control synthesis techniques, as well as analysis tools for establishing stability and performance guarantees.

Chemical engineers and engineers working in automotive industry may significantly benefit from this book, since we devote a whole chapter to the control of gas emissions in automotive catalysts. They are going to gain insight into the mechanisms due to which the current production strategies operate effectively, despite such strategies actually being heuristic.

Graduate classes in engineering and applied mathematics could also use this as a supplemental textbook. Reading parts of the book is a viable alternative to homework exercises or finals in classes such as nonlinear systems, nonlinear control, adaptive control (Sections 2.3, 3.1.3, 3.2.1), control of distributed parameter systems, robust control

(Chapters 7, 14), linear systems and linear control (Chapters 2, 3, 6, 7), and ordinary or partial differential equations with applications.

The reader is assumed to have a basic graduate-level background on differential equations and calculus. All required notions, such as Lyapunov stability, as well as basic inequalities and lemmas, such as Young's inequality and Barbalat's lemma, used in this book are summarized in appendices for the reader's convenience.

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