

# Preface

Singular perturbation theory plays a significant role in analysis and design of control laws. It presents control engineers with the advantage of model reduction that simplifies both the software and hardware implementation of control algorithms. Through this book, our goal is to present the reader with control design techniques that extend these benefits to a larger class of systems, specifically to those that are not traditionally controlled through this methodology. Our aim in doing so is to show the reader that some classes of nonminimum phase control problems can be actively controlled in real-time through use of singular perturbation methods.

The motivation of our work came from the realization that a large set of multiple time scale systems cannot be cast in the standard singularly perturbed form required for the results of singular perturbation theory to hold. Take for instance the large-scale systems we find around us today. A majority of these systems possess interacting phenomena that occur at widely different speeds. If these interactions and the associated systems are linear, then one can use either aggregation, block-diagonalization, or linear systems theory concepts [61, 9] to relate the wide spectrum of speeds to small quantities that are a function of the system parameters. The presence of these small quantities causes stiffness and high dimensionality in dynamic equations, which is remedied by singular perturbation techniques in the control literature. However, for nonlinear systems these small parameters are not only difficult to find, but also a function of the operating conditions. Recent research also indicates that these small parameters are not unique to a physical system and are an artifact of the choice of the underlying coordinate system [64]. Such dynamical models are called *nonstandard singularly perturbed systems*, and control of these systems is the focus of this book.

Nonstandard singularly perturbed systems are not uncommon. From aerospace vehicles to high-gain feedback systems, all exhibit multiple time scales and are nonstandard. In general, a dynamical model of any multiple time scale system developed using Hamilton's principle or Newton's laws of motion is in nonstandard form. From this viewpoint, we begin the book with a general discussion of multiple time scale phenomena in Chapter 1 and motivate the reader to follow our development of different examples to qualitatively understand this concept. The latter end of this chapter formally addresses the issue of modeling time scale systems in singularly perturbed standard and nonstandard forms. Our main modeling tool for casting multiple time scale systems in these forms is the forced singular perturbation method, and we provide the reader with the basic idea of this method throughout the book.

Chapter 2 revisits the essential concepts of singular perturbation theory. We introduce the necessary conditions for the model reduction results to hold in this chapter and provide also a geometric perspective for them. We do not provide proofs of the results detailed in this chapter, and we refer the reader to books [75, 98, 73] for rigorous details. Additionally, in this chapter we detail the composite Lyapunov function approach that

we employ throughout the book for analyzing the stability and robustness properties of the control algorithms designed.

In this book, nonlinear techniques to address two main control problems are presented. In the first control problem, we are interested in developing control techniques to ensure the closed-loop system asymptotically follows a desired slow state reference. To help motivate and elucidate the concepts of the new ideas, we first consider the two time scale counterpart of this problem. We further break down the problem to separately consider standard and nonstandard forms of singularly perturbed systems in Chapter 3 and Chapter 4, respectively. Chapter 3 revisits the composite control scheme for controlling standard forms of singularly perturbed systems, and our aim is to provide the reader with a slightly different perspective on this control scheme than the one presented in [51]. We motivate our discussion through simple examples and encourage the reader to see why this approach is widely employed. We detail its shortcomings and develop new results to avoid these in Section 3.2. This new approach is called *modified composite control*, and we show how a slight but important modification to the composite control scheme assists in controlling a larger class of standard singularly perturbed systems. We end this chapter with a discussion of both techniques, and open discussion for controlling nonstandard singularly perturbed systems. In Chapter 4 we develop a novel *indirect manifold construction approach* to counter some of the challenges that appear while controlling nonstandard singularly perturbed systems. Our primary goal here is to show how a control engineer can still take advantage of model reduction for systems that do not satisfy the essential conditions of singular perturbation theory. Our results are independent of the small parameter, and hence are valid for both standard and nonstandard forms of multiple time scale systems. Our presentation will allow us to consider control of these systems through different controller speeds, and this we discuss in Section 4.2. Finally, we move to Chapter 5 to extend the indirect manifold construction approach to systems with multiple time scales. The major contribution of these results is best detailed in Chapter 6, where we show application of these techniques to some benchmark nonminimum phase control problems.

The second control problem considered in this book requires us to develop control algorithms to ensure the closed-loop system follows both prescribed slow and fast state trajectories simultaneously. This presentation will be limited to two time scale systems represented in nonstandard form, and we develop a new algorithm in Chapter 7. The technique presented here is validated through simulation of a generic enzyme kinetic model and a high-fidelity nonlinear model of an F/A-18 HARV (High Angle-of-Attack Research Vehicle) aircraft model.

Throughout the book we introduce new ideas and discuss issues through tutorials and simple examples, after which we formulate and analyze the proposed control techniques for a general class of dynamical systems. We also discuss the benefits and the limitations of the techniques developed in the hope that these discussions will guide control engineers to judiciously choose from different control techniques presented. The book has been written in such a way that mathematical rigor progressively increases from one chapter to the next. It is important to point out that our book does not discuss how time scales can be characterized and when they are defined. Some of these research topics are still being studied and we refer the reader to details in [51, 64]. Throughout the book we assume that time scale phenomena exist for the systems we consider.

This book is written for researchers and practitioners in control engineering and applied mathematicians interested in control theory. It differs from the recent text by Ramnath [80] which focuses on linear aerospace systems. The methods and results presented in this book both generalize and extend the techniques in the book by Kokotović, Khalil

and O'Reilly [51], and Naidu [70] for a broader class of singularly perturbed systems. While this book is not a textbook in the traditional style, it can be used as a reference for an advanced control theory course for a first level graduate course on time scale methods in nonlinear control. Chapters 1, 2, and 3 and examples from Chapter 6 are also suitable for a first course on nonlinear control, or for reading/project assignments for undergraduates, while Chapters 4, 5, and 7 can be incorporated in a more advanced course on nonlinear feedback design.

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I dedicate this book to my wife, Stephanie, for her love, support, and steadfast encouragement in all of my endeavors.

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