

Contents

List of Figures	xiii
List of Tables	xix
List of Algorithms	xxi
Preface	xxiii
List of Notation	xxvii
I Theory	1
1 Introduction to Games	3
1.1 Introduction	3
1.2 Applications	3
1.3 Overview on different types of games	4
1.4 Nash equilibrium and dominant strategy	8
1.5 Cournot duopoly and iterated dominance algorithm	13
1.6 Stylized strategic models	14
1.7 Notes and references	16
2 Two-Person Zero-Sum Games	19
2.1 Introduction	19
2.2 Formalization as matrix games	19
2.3 From conservative strategies to saddle-points	20
2.4 From two-person zero-sum games to H^∞ -optimal control	23
2.5 Examples of two-person zero-sum games	24
2.6 Notes and references	26
3 Computation of Saddle-Points and Nash Equilibrium Solutions	27
3.1 Introduction	27
3.2 Graphical resolution: An example	27
3.3 Saddle-points via linear programming	33
3.4 Nash equilibrium via linear complementarity programming	35
3.5 Notes and references	40

4	Refinement on Nash Equilibrium Solutions, Stackelberg Equilibrium, and Pareto Optimality	41
4.1	Introduction	41
4.2	Refinement on Nash equilibrium solutions	41
4.3	Stackelberg equilibrium	44
4.4	Pareto optimality	47
4.5	Notes and references	49
5	Coalitional Games	51
5.1	Introduction	51
5.2	Coalitional games with transferable utility (TU games)	51
5.3	Game-theoretic examples of operations research problems	53
5.4	Imputation set	56
5.5	Properties	57
5.6	Cooperative differential games	57
5.7	Notes and references	59
6	Core, Shapley Value, Nucleolus	61
6.1	Introduction	61
6.2	Core	61
6.3	Shapley value	63
6.4	Convex games	65
6.5	Nucleolus	66
6.6	Notes and references	67
7	Evolutionary Game Theory	69
7.1	Introduction	69
7.2	Population of incumbents and mutants	69
7.3	Evolutionarily stable strategy, dominance, and equilibrium	71
7.4	Formal definition of evolutionarily stable strategy	73
7.5	Implications and examples	74
7.6	Notes and references	78
8	Replicator Dynamics and Learning in Games	79
8.1	Introduction	79
8.2	Replicator dynamics	79
8.3	Stationarity, equilibria, and asymptotic stability	81
8.4	Learning in games	82
8.5	Notes and references	85
9	Differential Games	87
9.1	Introduction	87
9.2	Optimal control problem	87
9.3	Differential game	90
9.4	Linear-quadratic differential games	93
9.5	H^∞ -optimal control as linear-quadratic differential game	94
9.6	Notes and references	96
10	Stochastic Games	97
10.1	Introduction	97
10.2	The model	97

10.3	A brief overview on applications	100
10.4	Two-player zero-sum stochastic games	101
10.5	The Big Match: “Work hard” or “enjoy life”	102
10.6	The Absorbing game: A variant of the Big Match	103
10.7	Other seminal results and further developments	104
10.8	Notes and references	105
11	Games with Vector Payoffs: Approachability and Attainability	107
11.1	Introduction	107
11.2	Approachability theory	108
11.3	A dual perspective: Connection with robust control	111
11.4	The concept of attainability	114
11.5	Conclusions and future directions	119
11.6	Notes and references	120
12	Mean-Field Games	121
12.1	Introduction	121
12.2	Formulating mean-field games	122
12.3	Existence and uniqueness	126
12.4	Examples	127
12.5	Robust mean-field games	130
12.6	Conclusions and open problems	138
12.7	Notes and references	138
II	Applications	141
13	Consensus in Multi-agent Systems	143
13.1	Introduction	143
13.2	Consensus via mechanism design	143
13.3	A solution to the <i>Consensus Problem</i>	146
13.4	A solution to the <i>Mechanism Design Problem</i>	147
13.5	Numerical example: Team of UAVs	151
13.6	Notes and references	154
14	Demand Side Management	155
14.1	Introduction	155
14.2	Population of TCLs	155
14.3	Turning the problem into a mean-field game	158
14.4	Mean-field equilibrium and stability	159
14.5	Numerical example	162
14.6	Notes and references	163
15	Synchronization of Power Generators	165
15.1	Introduction	165
15.2	Multi-machine <i>transient stability</i> in power grids	166
15.3	Modeling the transient as a mean-field game	169
15.4	Synchronization explained as stable mean-field equilibrium	172
15.5	Numerical example	175
15.6	Notes and references	180

16	Opinion Dynamics	181
16.1	Introduction	181
16.2	Opinion dynamics via local averaging with adversaries	183
16.3	Using <i>Blackwell's Approachability Principle</i>	186
16.4	<i>Consensus, polarization, and plurality</i> using contractivity and invariance	187
16.5	Numerical example	190
16.6	Notes and references	192
17	Bargaining	195
17.1	Introduction	195
17.2	Bargaining mechanism	196
17.3	Preliminaries: Nonexpansive projection and related bounds	197
17.4	Convergence of the bargaining mechanism	199
17.5	Numerical example	202
17.6	Notes and references	203
18	Pedestrian Flow	205
18.1	Introduction	205
18.2	Model and problem setup	206
18.3	Mean-field formulation with common cost functional	208
18.4	State space extension	209
18.5	Stability	212
18.6	Numerical example	214
18.7	Notes and references	216
19	Supply Chain	219
19.1	Introduction	219
19.2	Supply chain with multiple retailers and uncertain demand	220
19.3	Family of balanced games	221
19.4	Turning the repeated TU game into a dynamic system	223
19.5	Allocation rule based on feedback control synthesis	225
19.6	The Shapley value as a linear allocation rule	226
19.7	Numerical example	228
19.8	Notes and references	229
20	Population of Producers	231
20.1	Introduction	231
20.2	Production of an exhaustible resource	231
20.3	Robust mean-field equilibrium production policies	233
20.4	Stability of the microscopic dynamics	235
20.5	Stability of the macroscopic dynamics	236
20.6	Numerical example	237
20.7	Notes and references	238
21	Cyber-Physical Systems	241
21.1	Introduction	241
21.2	A model of CPS	242
21.3	Turning a CPS into a mean-field game	243
21.4	Humans in the loop and heuristic policies	245
21.5	Asymptotic stability	247

21.6	Numerical example	248
21.7	Notes and references	249
A	Mathematical Review	251
A.1	Sets and vector spaces	251
A.2	Normed linear vector spaces	251
A.3	Matrices	252
A.4	Convex sets and convex functionals	253
B	Optimization	255
B.1	Optimizing functionals	255
B.2	Mathematical optimization	256
C	Lyapunov Stability	259
D	Some Notions of Probability Theory	261
D.1	Basics of probability theory	261
D.2	Random vectors	262
D.3	Integrals and expectation	263
E	Stochastic Stability	265
E.1	Different definitions of stochastic stability	265
E.2	Some fundamental theorems	266
F	Indistinguishability and Mean-Field Convergence	271
	Bibliography	273
	Index	289