

Symbols, Acronyms, Index

Symbols

- $\langle \cdot, \cdot \rangle$, inner product
- \odot , Hadamard product
- \otimes , Kronecker product, see FAIR 2 (p. 17)
- $\| \cdot \|$, L_2 -norm and ℓ_2 -norm
- $| \cdot |$, Euclidian norm in \mathbb{R}^d
- α , regularization parameter (Section 6.3)
- β , stability parameter for Hessian (Section 6.3)
- η , edge parameter (Section 7.4)
- Ω , spatial domain, see FAIR 1 (p. 11)
- ρ , radial basis function (Section 5.3.1) or joint density (Section 7.3)
- σ , kernel width (Section 7.3)
- θ , scale-space parameter (Section 3.6)
- d , spatial dimension, see FAIR 1 (p. 11)
- H, \mathbb{H} , (approximative) Hessian of J ; entropy, cf. eq. (7.6); coarse grid
- h , cell size, discretization width (Chapter 3), fine grid
- $I = I_n$, see matrix, identity
- \mathbb{M} , regularization matrix (Section 6.3.2) or approximation to d^2D (Section 9.1)
- m , discretization size
- n , number of discretization points
- $n[\mathcal{T}]$, normalized image gradient (Section 7.4)
- \mathcal{O} , see Landau symbol
- omega, MATLAB representation of spatial domain Ω
- \mathbb{P} , see matrix, interpolation or prolongation operator
- \mathcal{Q}, Q , see basic function (Chapter 4)
- r_j , landmark in reference image (Chapter 5)
- t_j , landmark in template image (Chapter 5)
- u , displacement $u = y - y^{\text{ref}}$
- uc, current discretized displacement
- W , weighting matrix for spline regularization (Chapter 3)
- w, wc , coefficient vector for transformations (Chapter 4)
- w_{ref} , inhomogeneity for parameter regularization (Section 6.3.2)
- xc, current grid (Chapter 3)
- y , transformation, see FAIR 1 (p. 11)
- y^h, y_c , discretized transformation
- $y^{\text{ref}}, y_{\text{ref}}$, see regularization inhomogeneity (Section 8.2)
- derivative
 - dD, dD , of distance D
 - $d^2D, \text{d}2D$, approx. 2nd of D
 - dJ, dJ , of objective J
 - dS, dS , of regularizer S
 - $d^2S, \text{d}2S$, 2nd of regularizer
 - dT, dT , of template (Chapter 3)
 - dy, dY , of parametric transformation
- differential operator
 - Δ , Laplacian
 - $\nabla \cdot$, divergence

∇ , gradient operator
 ∇^h , discretized gradient operator
 ∇T , image gradient (Section 7.4)
 ∂ , ∂_j , partial
 ∂^h , ∂_j^h , discretized
 B , discretized partial differential operator (Section 8.3)
 \mathcal{B} , partial differential operator (Section 8.2)
distance
 \mathcal{D} , D , (Chapter 7)
 D_c , current value
 \mathcal{D}^{LM} , landmark, see FAIR 6 (p. 58)
 \mathcal{D}^{NCC} , normalized cross-correlation, see FAIR 13 (p. 99)
 \mathcal{D}^{NGF} , normalized gradient field, see FAIR 16 (p. 108)
 \mathcal{D}^{SSD} , L_2 -norm, see FAIR 9 (p. 71)
joint functional
 \mathcal{J} , see FAIR 1 (p. 11)
 J , J^h , discretized
 J_c , current value
reference
 \mathcal{R} , function, see FAIR 1 (p. 11)
 R_c , $R(\mathbf{x}_c)$, discretized (Chapter 3)
 R , coefficients for representation (Chapter 3)
 data_R , data (Chapter 3)
regularizer
 \mathcal{S} , S , (Chapter 8)
 S_c , current value
 \mathcal{S}^{TPS} , bending energy of a thin plate (Section 5.3.1)
template
 \mathcal{T} , template, see FAIR 1 (p. 11)
 T_c , $T(\mathbf{x}_c)$, discretized (Chapter 3)
 T , coefficients for representation (Chapter 3)
 data_T , data (Chapter 3)
 $T[y]$, transformed, see FAIR 1 (p. 11)

Acronyms

*c, current iterate
CAD, Canadian Acronym Disorder
CG, CG, conjugate gradient scheme
CT, computer tomography
FAIR, Flexible Algorithms for Image Registration
 \star^h , discretized
his, structure for iteration history, used in optimization schemes
HNSP, Human NeuroScanning Project
 ℓ -BFGS, limited memory Broyden–Fletcher–Goldfarb–Shanno scheme
LM, landmarks
MI, μI , Mutual Information, see FAIR 15 (p. 106)
ML, multilevel
MLdata, structure for multilevel data representation
MLIR, multilevel image registration
MLPIR, multilevel parametric image registration
NCC, ncc , Normalized Cross-Correlation, see FAIR 13 (p. 99)
NGF, ngf , normalized gradient field, see FAIR 16 (p. 108)
NPIR, nonparametric image registration
para, collection of intermediate variables, used for visualization
PDE, partial differential equation
PET, positron emission tomography
SPECT, single photon emission computed tomography
PIR, parametric image registration
ref, $\star\text{Ref}$, reference, used as inhomogeneity in regularization
SSD, ssd , Sum of Squared Differences, see FAIR 9 (p. 71)
TPS, tps , thin-plate spline

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