Preface

Due to their utility and broad applicability in many areas of applied mathematics and physical science (e.g., computerized tomography, Navier–Stokes equations, pattern recognition, and image restoration, among others) the Method of Alternating Projection (MAP) continues to receive significant attention. The main purpose of this book is to describe and analyze all available algorithms in the MAP family for solving the general problem of finding a point in the intersection of several given sets that belong to a Hilbert space. Different types of algorithms and different applications are studied for the following types of sets: subspaces, linear varieties, and general convex sets. The second goal of this book is to unify all these algorithms into a common framework.

In recent decades, many papers have appeared dealing with the MAP family and especially its wide applicability. Four important books have also appeared that dedicate one or more chapters to the topic of MAP: The book by Censor and Zenios [65], *Parallel Optimization*, which is mainly concerned with "row-action" methods that are suitable for parallel architectures; the book by Deutsch [95], *Best Approximation in Inner Product Spaces*, that dedicates one chapter to this topic, paying significant attention to Dykstra's algorithm for finite-dimensional inner product spaces; and the book by Stark and Yang [233] *Vector Space Projections: A Numerical Approach to Signal and Image Processing, Neural Nets, and Optics* and the book by Byrne [49] *Applied Iterative Methods*, both which describe important problems in various areas of science and engineering that can be solved using MAP.

Nevertheless, much of the inspiration that motivated us to write this book comes from a fifth publication on a related topic, the classical book by Luenberger [190], *Optimization by Vector Space Methods*, which, according to its author has as a primary objective "to demonstrate that a rather large segment of the field of optimization can be effectively unified by a few geometric principles of linear vector space theory." We share this thought. For this reason, the support pillar of the approach taken in this book is the geometry associated with Hilbert spaces.

This book grew up from our personal notes while teaching the topic of MAP in several graduate courses, and also advanced undergraduate courses, in different universities: Universidad Central de Venezuela (Caracas, Venezuela) and Universidad Simón Bolívar (Caracas, Venezuela) for the last 12 years, and at Universidad Nacional del Sur (Bahía Blanca, Argentina) once in 1999. Therefore, it evolved as a textbook for advanced undergraduate or first year graduate students. However, since the book is comprehensive, it can also be used as a tutorial or a reference by those researchers who need to solve alternating projection problems in their work. The required background is some familiarity with matrix algebra and numerical analysis.

Throughout the book we have used a standard notation as similar as possible to that found in the previous books mentioned above. In Chapter 1 we present a list of some applications of MAP to problems of practical interest in science and engineering. In Chapter 2 we present a review of the vector spaces used in the rest of the book, including a list of basic concepts and results in functional analysis. From that point on, the chapters are introduced following a chronological order. Chapter 3 introduces MAP on subspaces. Here we follow the classic approach on this topic, beginning with a theoretical framework as it was originally developed by von Neumann [209] and Halperin [148], and followed by a treatment on the rate of convergence of MAP, and some acceleration techniques. In Chapter 4 we study the row-action methods, which are specially designed for linear varieties. We introduce also a recent scheme of acceleration, and we close with a discussion on the more general convex feasibility problem. In Chapter 5 we show the very important Dykstra's theorem, which is a skillful extension of MAP to the convex case, and we discuss its rate of convergence in the polyhedral case and also the delicate issue of stopping the convergence process for Dykstra's method. In Chapter 6 we describe two distinct applications of MAP whose variables are matrices. The first is related to a least-squares approach for solving some special problems in statistics and mathematical economy. The second is related to the model updating problem that appears in the design of vibration structural dynamic systems. We wanted to keep the book as short as possible without sacrificing the clarity of the exposition. Thus, at the end of every chapter, we have included some information under the heading *Comments and Additional References* to include the historical perspective and to list in a condensed form the more advanced topics and the ongoing lines of research. At the end of each chapter we also offer a variety of problems to be solved. Some of them are closely related to the theoretical development of the alternating projection field, and others are related to the practical aspects of the described algorithms.

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