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Lectures on BSDEs, Stochastic Control, and Stochastic Differential Games with Financial Applications. *By René Carmona.* SIAM, Philadelphia, 2016. $84.00. x+265pp., softcover. ISBN 978-1-61197-423-2. https://doi.org/10.1137/1.9781611974249.

This book is a collection of lecture notes on various topics in the area of stochastic control, stochastic differential games, and financial applications. It makes for an excellent resource for those who already have some familiarity with these topics, although, as the author points out, it is not a polished textbook.

If the reader is not well versed in stochastic processes and control theory, this book may not be the best introduction since prerequisite knowledge of these topics is a must before diving into the material. On the other hand, the book contains a very useful section of notes at the end of each chapter describing references that can be used to get up to speed and to put the material into the wider context. In this respect, it can be used as a guide for those interested in navigating the various books and papers that are drawn upon.

The book is divided into three parts: "Stochastic Calculus Preliminaries," "Stochastic Control," and "Stochastic Differential Games." The underlying theory is developed in a systematic manner, and many financial applications are scattered throughout the book.

While the material in the first two parts builds up to and supports the final part on stochastic games, the last part is self-contained and can be studied in isolation by those already familiar with stochastic control. The preceding material can then be referred to as necessary.

The first two chapters are in Part 1, "Stochastic Calculus Preliminaries." Chapter 1 contains background material on stochastic differential equations (SDEs), in which existence and uniqueness theorems are proved for SDEs of the form $dX_t = b(t, X_t)\,dt + \sigma(t, X_t)\,dW_t$, where $\mathbf{W}$ is Brownian motion. Both stochastic and deterministic coefficient versions are considered. A common pattern in the book is that a general problem is first introduced and then more specific versions of the problem are studied as the chapter progresses. Chapter 2 introduces backward stochastic differential equations (BSDEs), mean-field BSDEs, reflected BSDEs (RBSDEs), and forward-backward SDE (FBSDEs). Existence and comparison theorems are presented and proved. This chapter also contains the book's first application, pricing European options and pricing and hedging American contingent claims.

Part 2, "Stochastic Control," contains Chapters 3 and 4. In Chapter 3 the stochastic control problem is studied, where the SDE coefficients also depend on a control process α. Several applications are described including: greenhouse gas emission control, classical Merton portfolio optimization, Markowitz mean-variance portfolio selection, commodity inventory valuation, and high water marks and hedge fund management. Chapter 4 describes probabilistic approaches to stochastic control, where the connection between the stochastic control problem of Chapter 3 and the theory of BSDEs in Chapter 2 is made.

Part 3, "Stochastic Differential Games," contains Chapters 5 and 6. Chapter 5 defines the setup in which a set of N players can take actions $\alpha = \left(\alpha^1, \ldots, \alpha^N\right)$ at any point in time, and the state of the system is described by $dX_t = b(t, X_t, \alpha_t)\,dt + \sigma(t, X_t, \alpha_t)\,dW_t$. A special case where there is only one player is equivalent to the stochastic control problem described in Chapter 3. After the cost functional for the problem is defined, the author defines the various types of equilibria considered: Nash equilibrium, open loop Nash equilibrium (OLNE), deterministic Nash equilibrium (DNE), closed loop Nash equilibrium (CLNE), and closed loop Nash equilibrium in feedback form (CLFFNE). These definitions are made in terms of constraints on the set of player actions. The chapter then examines several special cases and develops the corresponding theory. The last section in the chapter covers a predatory trading game model.

The sixth and final chapter of the book covers another special class of games called

mean-field games. In this class, the players are assumed to have similar behavior. Several applications are presented as well as open problems. As with all of the chapters, the notes section at the end of this chapter is filled with references for the material, including the techniques and applications. One of these references is for a two-volume book by the author, *Probabilistic Theory of Mean Field Games with Applications* (Springer, 2017).

Overall, the book offers a taste of financial applications and will appeal to those who are motivated by applications and interested in mastering the underlying theory. In addition, it can be viewed as a supplementary resource for the study of stochastic control and stochastic games.

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