Preface

This book grew out of the lecture notes I prepared for a graduate class I taught at Princeton University in 2011–12, and again in 2012–13. My goal was to introduce the students to stochastic analysis tools, which play an increasing role in the probabilistic approach to optimization problems, including stochastic control and stochastic differential games. I had invested quite a bit of effort in trying to understand the groundbreaking works of Lasry and Lions on mean field games, and of Caines, Huang, and Malhamé on Nash certainty equivalence. These initial results were intriguing, and definitely screaming for a probabilistic interpretation. While the tools of optimal control of stochastic differential systems are taught in many graduate programs in applied mathematics and operations research, I was intrigued by the fact that game theory, and especially the theory of stochastic differential games, are rarely taught in these programs. In fact, I was shocked by the lack of published literature in book form, a sad state of affairs which prompted me to write lecture notes for the class.

This would have been the end of the story if I hadn’t shared some of my notes with a friend and co-author of mine, Jean-Pierre Fouque, who decided to convince me to turn these lecture notes into a book. His perseverance, together with my desire to help those applied mathematicians trying to learn the theory of stochastic differential games despite the lack of sources in textbook form, helped me to find the time to clean up my original class notes. Still, this short preface should contain a clear disclaimer emphasizing the fact that the present manuscript is more a set of lecture notes than a polished and exhaustive textbook on the subject matter. Many experts on the subject could produce better treatises. But until then, I hope these notes will be helpful to young researchers and newcomers to stochastic analysis, eager to understand the subtleties of the modern probabilistic approach to stochastic control and stochastic differential games.

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Important Aside. The royalties generated by the sales (if any) of this book will be used, in their entirety, to fund a student prize awarded every other year during the SIAM Conference on Financial Mathematics.

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