Preface

This book is an introduction to a family of discontinuous Galerkin (DG) methods applied to some steady-state and time-dependent model problems. A special effort was made to have the material self-contained as much as possible. The book is well suited to numerical analysts interested in DG methods but also to applied mathematicians who study CFD or porous media flow. Practical implementation issues are discussed, which can be of interest to engineers. The material can be used in a graduate level course on the numerical solution of partial differential equations. Chapter 1 is introductory and can be used in a scientific computing class for senior undergraduate students. Prerequisites are calculus and linear algebra.

In this book, we mainly focus on the class of *primal* DG methods, namely variations of interior penalty methods. In the text, these methods are referred to as the symmetric interior penalty Galerkin (SIPG), incomplete interior penalty Galerkin (IIPG), and nonsymmetric interior penalty Galerkin (NIPG) methods. The book is divided into three parts: Part I focuses on the application of DG to second order elliptic problems in one dimension first and then in higher dimensions. In Part II, the time-dependent parabolic problems (without and with convection) are presented. Finally, Part III covers some applications of DG to solid mechanics (linear elasticity), to fluid dynamics (Stokes and Navier–Stokes), and to porous media flow (two-phase and miscible displacement).

We try to discuss both theoretical and computational aspects of the DG methods. In particular, for the elliptic equations, a code written in MATLAB® for one-dimensional problems is provided in Appendix B.1. The text contains algorithms for the implementation of DG methods in two or three dimensions. Corresponding routines written in C are provided in Appendix B.2 as well.

One objective of this book is to teach the reader the basic tools for analyzing DG methods. Proofs of stability and convergence of the method are given with many details. Another objective is to teach the reader the coding issues of DG methods: data structure, construction of local matrices, and assembling of the global matrix. Several computational examples are provided. Finally, by presenting specific applications of DG to important engineering problems, we hope to convince the reader that the DG method is a competitive approach for solving his/her own scientific problem.

The first DG methods were introduced for hyperbolic problems, which we do not cover in this book. The treatment of DG methods for conservation laws can itself be the object of an entire book. Other important topics not discussed in this book include solvers and preconditioning.

This book stems from a collection of notes that I used in several graduate classes while I was teaching at the University of Pittsburgh. I would like to thank all the students for their feedback and comments. Some of the work presented in this book was funded by the National Science Foundation. I am grateful for their support.

B.M. Rivière