Preface

The huge impact of mathematics on modern society is undoubtedly due to the power of mathematical modeling. Mathematics has not only an unreasonable effectiveness in the natural sciences, as concluded by Wigner already long ago, but it also contributes to other fields in a remarkably effective way. This implies that mathematical modeling - the ability to apply mathematical concepts and techniques to real life systems - has considerably expanded over the last decades. It is impossible to deal with its many and various aspects in one course or one textbook. In the present text we restrict ourselves to applications in the natural sciences and focus on an advanced level of modeling. The present book has grown out of lecture notes of a course for advanced undergraduates at the University of Twente. The original material, aiming at a ten weeks course, is greatly extended with extra topics and many examples to provide lecturers the opportunity to select those parts that fit the audience best. This book is intended to be used as a textbook but, as we hope, it will also be useful as a mere source of reference and inspiration for students and researchers alike.

Teaching mathematical modeling is a quite complicated challenge. On the one hand one has to expose a great variety of general mathematical concepts, and on the other hand one has to treat the principles of the field of application in some detail. It is this diversity of applicable techniques and possible applications that could seduce an author to present the subject as a long series of ingenious case studies, in which students can hardly discover any coherence. It could even disappoint him or her, since having digested many particular models does not guarantee that one knows how to proceed when confronted with a new situation. To convince students of the power and beauty of modeling, we offer in this book an extensive exposition of general principles. Since students gain most of a course if its structure is clearly highlighted, most chapters are devoted to central issues such as dimensional analysis, conservation principles, balance laws, constitutive relations, stability, robustness, and variational methods. The core of these chapters will form the backbone of any course on mathematical modeling.

This book aims at applications of modeling techniques, and we think that the relevant ideas and techniques are best presented via examples and exercises. The book contains a multitude of class room examples and exercises throughout the text and at the end of several chapters a section with 'challenging problems' has been added. Furthermore, the last chapter is fully devoted to extensively worked out case-studies. The examples mostly stem from classical mechanics, wave phenomena and continuous mechanics, showing the background of the authors. However, this does not imply that the book could not be used to study the mathematical modeling of topics from other disciplines. On the contrary, we have tried to keep the treatment of modeling principles as general as possible.

Chapter 1 is devoted to dimensional analysis and scaling. It is fascinating to show how strong conclusions can be drawn about a system just by looking at the physical dimensions of the relevant quantities. Our hope is that the presented examples are so convincing that the reader will never start any modeling activity without first checking whether these techniques might be applied.

In Chapter 2 we introduce some basic elements of modeling, namely conservation principles and constitutive relations. These notions are so general that any modeler must master them. They are first introduced in one dimension, so that the reader gets familiar with them in a natural way. The so-called Transport Theorem plays a central role in generalizing them to more dimensions. This theorem allows us to deal with all kinds of quantities that may be transported (scalar, vector, and tensor like) on an equal footing.

In Chapter 3 we summarize the basics of differential equations. In our manner of presentation the analogies rather than the differences between ordinary and partial differential equations are emphasized.

Chapter 4 deals with stability and robustness. Stability is an essential aspect of any model analysis, since most models are used to control the systems under consideration. The related concept of robustness is important, since it provides the modeler with information about the sensitivity of the model with respect to perturbations of its parameters.

In Chapter 5 variational methods are discussed. These methods deserve an extensive treatment, since remarkably many problems in nature can be put in variational form, i.e. can be formulated as an optimization problem. We also point out how a variational formulation may yield a useful guideline how to calculate the solution of a model numerically.

Chapter 6 is meant as a summarizing show-case, in which the ideas and techniques in the preceding chapters are applied to real-life problems. To that aim we extensively work out four advanced examples. The first one deals with polymer dynamics. It nicely shows how a modeler may benefit by dimensional analysis. The second example concerns fiber spinning. It shows how a relatively simple system can lead to a hard stability analysis. The third example shows the modeling of water waves. It demonstrates the power of a variational approach. Eventually, in the fourth example we study the transmittance of light through an optical fiber.

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E. van Groesen
J. Molenaar