Preface

Welcome! This book leads you on an introduction into the fascinating realm of financial mathematics and its calculus. Modern financial mathematics relies on a deep and sophisticated theory of random processes in time. Such randomness reflects the erratic fluctuations in financial markets. I take on the challenge of introducing you to the crucial concepts needed to understand and value financial options among such fluctuations. This book supports your learning with the bare minimum of necessary prerequisite mathematics.

To deliver understanding with a minimum of analysis, the book starts with a graphical/numerical introduction to how to adapt random walks to describe the typical erratic fluctuations of financial markets. Then simple numerical simulations both demonstrate the approach and suggest the symbology of stochastic calculus. The finite steps of the numerical approach underlie the introduction of the binomial lattice model for evaluating financial options.

Fluctuations in a financial environment may bankrupt businesses that otherwise would grow. Discrete analysis of this problem leads to the surprisingly simple extension of classic calculus needed to perform stochastic calculus. The key is to replace squared noise by a mean drift: in effect, $\mathrm{d}W^2 = \mathrm{d}t$. This simple but powerful rule enables us to differentiate, integrate, solve stochastic differential equations, and to triumphantly derive and use the Black–Scholes equation to accurately value financial options.

The first two chapters deal with individual realizations and simulations. However, some applications require exploring the distribution of possibilities. The Fokker–Planck and Kolmogorov equations link evolving probability distributions to stochastic differential equations (SDEs). Such transformations empower us not only to value financial options but also to model the natural fluctuations in biology models and to approximately solve differential equations using stochastic simulation.

Lastly, the formal rules used previously are justified more rigorously by an introduction to a sound definition of stochastic integration. Integration in turn leads to a sound interpretation of Ito's formula that we find so useful in financial applications.

Prerequisites

Basic algebra, calculus, data analysis, probability and Markov chains are prerequisites for this course. There will be many times throughout this book when you will need the concepts and techniques of such courses. Be sure you are familiar with those, and have appropriate references on hand.

Computer simulations

Incorporated into this book are MATLAB/SCILAB scripts to enhance your ability to probe the problems and concepts presented and thus to improve learning. You can purchase MATLAB from the Mathworks company, http://www.mathworks.com. SCILAB is available for free via http://www.scilab.org.

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