Index

ϵ-chain, 229
ϵ-chain pseudo-orbit, 81, 228
ϵ-separated sets, 284
$\gg$ (exceedingly larger than), 87
μ-almost invariant, 62
σ-algebra, 44
σ-finite measure space, 44

ABC flow, 255
absolutely continuous, 6
absolutely continuous conditional invariant measure (ACCIM), 145
absolutely continuous densities, 8
absorbing Markov chain, 143, 145, 223
absorbing sets, 143
absorbing states, 143
absorption probabilities, 143
ACCIM, *see* absolutely continuous conditional invariant measure
action-based methods, 220
adaptive Delaunay tessellation, 322
additive noise, 51
additive reversibilization, 117
additive stochastic perturbation, 51
adjacency matrix, 29, 81, 182
adjacency matrix of a graph
    definition, 182
adjoint property, 47, 64
advecting forward in time, 253
advection of orbits, 48
algebraic and geometric multiplicities, 86
algebraic connectivity, 138
algebraic topology, 200
almost all, 207
almost-block-diagonal decomposition, 113
almost-block-diagonal form, 110, 111
almost every, 207
almost-invariance, 61, 64, 101
    weak transport descriptive of, 103
almost-invariant partitioning, 120
almost-invariant set, 61, 317
almost reducible graph, 104
alphabet, 270
Amazon book example, 133
Anosov diffeomorphism, 35
Antarctic polar vortex, 253
approximate invariant measure, 64
approximating the gradient, 247
approximation of the Frobenius–Perron operator, 319
area flux, 222
area-preserving, 65
    definition, 206
area-preserving flow, 147
arithmetic coding, 269, 271
Arnold tongues, 233
ASCII, 266
associated system, 57
assumed prior knowledge, 266
ASY chaos, 168
asymptotically stable orbit, 168
atomic invariant measure, 54
attracting Lagrangian coherent structure, 259
attractor, 16
austral winter, 253, 256
autonomous, 17

autonomous double gyre velocity field, 250
autonomous Hamiltonian system, 177
autonomous vector field, 18

Baker map, 331
balanced cut via spectral partitioning, 124
Banach space, 19, 31, 68
barriers, 198
  definition, 200
barriers to transport, 24
basin structure, 80
basis function, 69, 77
Bernoulli fullshift, 192
Bernoulli left shift grammar, 209
Bernoulli mapping, 286
Bernoulli shift, 163
Bernoulli shift map, 155, 164, 174, 207
betweenness centrality, 133
bi-differentiable, 14
bi-infinite, 163
bi-infinite sequences, 207
bilinear map, 33
Birkhoff ergodic theorem, 6, 60, 238, 296
Birkhoff ergodic theorem and histogram, 7
bisection method of the graph, 138
bistable dynamical system, 83
bistable system, 83, 226
bit rate, 268
  average, 268
block-diagonalizable stochastic matrix, 108
Boltzmann's ergodic hypothesis, 60
bookie, 267
Borel σ-algebra, 49
Borel sets, 44
bounded orbit, 168
Breadth First Search algorithm, 133
Bruijn graph, 177, 182, 183

C^∞ Riemannian manifold, 35
canonical ensemble, 60
Cantor, 229
Cantor removal process, 141
Cantor set, 54, 60, 141, 174, 228, 331
  cardinality of the continuum, 165
  closed, 165
  definition, 165
  generalized, 166, 209, 210
  invariant, 207, 208
  middle thirds, 209
  of measure zero, 103, 174
  of tongues, 233
  perfect, 165
  topological, 209
  totally disconnected, 165
  unstable chaotic saddle, 210
Cantor subset, 155
cardinality, 181
Cauchy–Green deformation tensor, 244
Cauchy–Green tensor, 260–262
ceil function, 294
cell-to-cell mapping, xi, xii
centrality measures, 133
chain rule of entropies, 273
change of coordinates, 163
change of variable formula, 46
channel capacity, 276, 280
channel coding theorem, 276
chaos
  controlling, 150
  fully developed, 12
  mechanisms of, 150
chaos theory
  applications of, 166
chaotic attractor, 14, 171
chaotic dynamical system, 8, 166
  information theoretic aspects of, 276
  paradox of continuity, 202
chaotic mixing, 243
chaotic orbit, 168
chaotic oscillators
  as information sources, 280
chaotic saddle, 60, 174, 292
chaotic transport, 200
chaotic transport across barriers, 206
characteristic function, 69, 76
childhood disease, 224
circle map, 59
classical optical flow, 48
Clauraut theorem, 33

clustering, 132
coarse partition may represent the transfer entropy, 302
coarse partition of rectangles, 188
coarse-grain estimation, 85
cobweb diagram, 14, 101–103
codimension-1, 17
coding theory, 179
coherence ratio, 121
coherent pair, 121
coherent set, 120
coherent structure, 21
communication system, 276
community finding, 132
community methods, 132
community structure, 80, 83, 132
commutes, 155
commuting diagram, 163
completely reducible, 82
completely reducible graph, 101
completely reducible system, 108
complex plane, 89
component, 81
component of a graph, 81
compound event process, 272
compression, 268
computational complexity, 327, 331
computer-validated proof of symbolic dynamics, 286
conditional entropy, 272, 300
conditional probabilities, 273
conjugacy, 12, 152, 207
  definition, 155
continuity eigenvalues, 108
continuity equation, 65
continuity equation in fluid mechanics, 48
continuous continuity equation, 48
continuous functions that converge uniformly, 94
continuous semigroup, 45
  of contractions, 46
continuous stochastic perturbation, 49
continuous tangent space, 35
continuous time process, 43
continuous unimodal maps, 99
continuously differentiable, 32
contraction mapping theorem, 155
convergence, 94
convex combination, 54
Courant–Fischer theorem, 115, 317
covering geometry, 329
C^r diffeomorphism, 33, 176
cross entropy, 273, 274
croupier, 169
cutting edges, 133

D-ary alphabet, 270
dark star, 104
definitions of chaos, 180
degree of surprise, 271
Delaunay tessellation, 318
Delaunay triangulations, 309
dendrogram, 137
dense periodic orbits, 168
density, 6
density of edges, 80
density of Markov map, 100
Devaney chaos, 167
Devaney definition of chaos, 286
devil's staircase, 186, 190
devil's staircase function, 14, 186, 232
devil's staircase topological entropy function, 233, 234
DHT, *see* distinguished hyperbolic trajectory
diagonalizing similarity transformation, 109
diffeomorphism, 14
  definition, 157
diffeomorphism of the plane, 155, 157
difference quotient, 19
differentiable dynamical system, 19
differentiable submanifold, 37
differentiable topological space, 35
directed graph, 29, 133
  definition, 180
discrete continuity equation, 8
discrete space description, 150
discrete time mapping, 18
  invertible, 32
  noninvertible, 32
discrete time stochastic process, 91
dissipation, 30, 56

distinguished hyperbolic trajectory (DHT), 41
divergence-free vector field, 48
dominant eigenvalue, 107
dominant eigenvector, 54, 143
dominant transport barrier, 128
double gyre system, 252
double-well potential, 15
  Duffing, 16
doubly stochastic, 114, 116
Duffing equation, 15, 248
Duffing flow, 174
Duffing map, 174, 176
Duffing oscillator, 16
Duffing oscillator Poincaré mapping, 175
dyadic, 191
dyadic graph presentation, 185
dyadic misplacements, 192
dyadic rational, 234
  definition, 235
dynamical system, 4, 30, 31
  closed relative to the window, 140
  coarse-grained representation of, 193
  decomposing into almost-invariant sets, 132
  invariant set of, 54
  stochastically perturbed, 224
  through information theory, 281
dynamically closed relative to a window, 141
dynamically open relative to a window, 141

edge betweenness methods, 133
edge of chaos, 227
edges, 133
eigenvectors wobble problem, 109
elementary permutation, 107
elements, 68
embedded horseshoe, 172
embedding, 68, 292
empirically estimating Lyapunov exponents, 238
encoding $c(x)$ for a random variable, 269
encoding extension, 270
ensemble density, 77
ensemble perspective, 2, 58
ensembles of trajectories, 48
entrance lobe, 213, 217
entrance region, 198
entropy, 150, 191
  by Markov model of the Ikeda map, 291
  by periodic orbits of the Ikeda map, 288
  variational principle for, 284
entropy coding, 265
entropy function, 233
entropy of the process, 271
entropy production, 302
entropy rate, 275
  average conditional, 283
equivalence relationship, 14, 155, 157
ergodic behavior, 60
ergodic hypothesis, 60
ergodic invariant density
  logistic map, 60
ergodic invariant measure, 59
ergodic theory, 58
ergodic-typical, 8
ergodicity, 21
  unique, 93
error probability of a decoder, 276
error rate, 276
escape, 214, 228
escape matrix, 222
escape rate, 103, 145, 149, 225
Escape Rate and the Hole, 143
Euclidean norm, 87
Eulerian PDF method, 66
Eulerian perspective, 2
European Centre for Medium-Range Weather Forecasts, 253
evolution of density, 8
evolution operator point of view, 43
existence, 9
existence and uniqueness theorem, 19
exit lobe, 213, 217
exit region, 198
exponential dichotomy, 39
exponential growth of error, 239

($\mathcal{F}$-)measurable function, 49
factor
  definition, 291
Fano's converse, 276
fast-slow system, 56
Feigenbaum diagram, 60, 183
Fiedler vector, 138
finite Markov chain, 69, 223
finite memory property, 72
finite rank transfer operator, 76
finite state Markov chain, 92
finite subspace of basis functions, 309
finite time Lyapunov exponent (FTLE), 7, 238
  as hyperbolic LCS, 263
  backward, 249, 262
  backward-time, 128
  fields, 243
  forward, 249
  forward-time, 128
  nonattracting ridges, 262
  of the Duffing equation, 249
  ridges, 128
  ridges in polar vortex, 256
finite time relative coherence, 145
finite time relatively coherent sets, 148
finite-dimensional Galerkin representation, 286
finite-sized transition matrix, 286
first passage time, 143
Floquet multiplier, 193
flow, 30, 31
fluid flow in the Gulf of Mexico, 72
flux, 214
flux across barriers, 218, 220
flux between basins, 225
Fokker–Planck equation, 49
Fokker–Planck operator, 193
follower-set construction, 183
forward (backward) lifetime escape function, 229
Fourier modes, 31
Fréchet derivative, 32
Fréchet differentiable, 32
Frobenius norm, 104
Frobenius–Perron fixed point equation, 52
Frobenius–Perron matrix, 78, 87
  approximation of, 309
Frobenius–Perron operator, xii, 1, 8, 9, 43, 44
  as push forward, 64
  for deterministic evolution of density in maps, 9
  of the logistic map, 10
  of the tent map, 11
  spectrum of, 66
Frobenius–Perron transfer operator, xi
FTLE, *see* finite time Lyapunov exponent
FTLE for the Gulf of Mexico, 229
full family of maps, 184
full tent map, 232
fullshift, 12, 154, 155, 190
fullshift horseshoe, 176
fullshift space, 155
fullshift symbolic dynamics, 55
functional equation, 52
fundamental matrix solution, 242
fundamental theorem of calculus, 9
future itinerary, 193
fuzzy inclusion, 116

GAIO, 314, 319
Galerkin method, 2, 67
Gedanken-experiment, 108
generalized eigenvalue problem, 125
generalized Rayleigh quotient, 125
generating partition, 75, 152, 187, 188, 190, 288, 289
  purpose of, 189
genericity, 14
Genus, 200
geodesic paths, 133
geometric convergence, 88
geometric multiplicity, 109
geometry of the linearized flow map, 259
Gersgorin disc theory, 108
Gibbs, J. W., 60
global attractor, 2
global climate, 274
global climate network
  mutual information mapping of, 308

global transport, 80
gradient of the flow map, 247
gradient system, 16
grammar, 12
  finer restrictions of, 181
  of the symbolic dynamics, 183
  restricted, 153
graph
  connected, 81
graph action, 22
graph network, 137
graph partitioning, 2
Gray-code order, 183, 185
  in the symbol space, 184
Grobman–Hartman theorem, 202
Gronwall inequality, 327, 328
group action, 30
group addition property, 30
group isomorphic, 30
growth of the lobes, 204
Gulf of Mexico, 141
  transport in, 225
Gulf of Mexico Deepwater Horizon oil spill disaster, 23
Gulf Stream, 23, 72

Hadamard–Perron manifold theorem, 37
Hamiltonian, 147
Hamiltonian system, 206
  area-preserving, 211
Harris chain, 93
Hartman–Grobman theorem, 32, 201
Henon map, 174, 175, 190, 286
  area-preserving, 205, 210, 213, 214
  generating partition of, 76
  homoclinic tangle of, 211
  invariant set of, 175
Henon mapping, 2
Henon-type maps
  decomposition of, 157
Hessian matrix of the FTLE field, 257
heteroclinic cycles, 210
heteroclinic orbit, 37, 38
heteroclinic orbit segment, 218
heteroclinic point, 38, 202
hierarchical relative coherence in the Gulf of Mexico, 228
higher-dimensional Markov partition, 187
Hilbert space, 31, 67, 68
  $\mathcal{H}(D)$, 95
histogram, 5
hitting times, 143
hole in the tent map, 235
hole-like nature of this dynamics, 234
homeomorphism, 12, 14, 73, 98, 155, 157, 165, 292
  as change of coordinates, 165
  continuous, 12
  definition, 155
  one-one, 12
  onto, 12
homoclinic connection, 179
homoclinic orbit, 38, 200, 243
  horseshoe on, 208
homoclinic point, 38, 202
homoclinic tangencies, 288, 294
homoclinic tangle, 38, 177, 206, 212, 213, 215, 292
homology theory, 155
horseshoe, 207
horseshoe dynamics, 198
horseshoe escape, 207
horseshoe map, 160
  inverse of, 159
horseshoe map decomposition, 158
horseshoe mapping, 157
horseshoe theory, 152
Huffman code, 266, 267, 270, 271
Huffman coding, 266, 268
Huygens–Steiner parallel axis theorem, 326
HYCOM model, 25, 228
hyperbolic
  definition, 201
hyperbolic fixed point, 32, 36
hyperbolic invariant set, 36
hyperbolic linear, 202
hyperbolic periodic points, 36
hyperbolic splitting, 36
hyperbolic structure, 49
hyperbolic trajectory, 39, 41
hyperbolicity, 34
  uniform, 34

idealized betweenness centrality, 134
idealized stratospheric flow, 127
identity property, 30
Ikeda map, 76
Ikeda–Hammel–Jones–Moloney attractor, 289, 290
imbedding
  definition, 292
in-degrees, 135
inappropriate partition, 152
incomplete folding, 177
incomplete horseshoes, 177
incompressible fluid, 48
incorrect partition, 182
indecomposibility, 63
independence, 275
independent
  definition, 273
independent and identically distributed (i.i.d.), 275
  stochastic process, 275
independent random variables, 273
infinite itinerary, 151
infinite symbol sequence, 154
infinite symbolic strings, 155
infinite-dimensional dynamical system, 47
infinitesimal comparison, 14
infinitesimal generator, 46
  for the semigroup of Frobenius–Perron operators, 48
infinitesimal operator, 45
  of Frobenius–Perron, 45
  of Koopman, 45
infinitesimal operator of Frobenius–Perron, 45
infinitesimal operator of Koopman, 45
infinity norm, 87
information channel capacity, 276
information expectation, 271
information flow, 298
information theory, 179, 265
information-bearing stochastic process, 281
infrared NH3 laser data, 152
initial conditions
  ensemble of, 5, 8, 20
inner product, 68
inspection of a dynamical system, 104
instantaneous code, 270
intermediate value theorem for kneading sequences, 99
invariant Borel measure, 59
invariant density, 9, 16, 52, 99
invariant manifold, 2
invariant measure, 6, 52, 288
  dominant, 10
invariant regions, 252
invariant set, 36, 141
invariant set in a flow, 55
invariant set in a map, 55
invariant set of the horseshoe, 208
invariant tori, 212
irrational rotation, 59
irreducible components, 107
irreducible graph, 82
irreducible matrix, 81, 144
irreducible subshifts of finite type, 284
isentropic surface, 243, 254
isolated state, 114
itineraries, 150
itineraries through the partition, 161

Jacobian matrix, 33
Japanese samurai sword, 208
jet core, 127
join, 75
  between two partitions, 283
  set theoretic, 283
joint entropy, 272, 275, 299
joint probabilities, 267, 272
joint probability density function, 272
Jordan curve, 201
Jordan curve theorem, 197

KAM islands, 252
KAM theory, 212
kernel form of the operator, 70
kneading sequence, 96
kneading theory, 156, 183–185, 189
Kolmogorov complexity, 267
Kolmogorov–Sinai entropy, 281, 282
Koopman operator, 45, 47
  as pull back, 64

isometric, 65
Kraft inequality, 270
Krylov–Bogoliubov theorem, 59
KS entropy, *see* Kolmogorov–Sinai entropy
Kullback–Leibler divergence, 273, 274, 299, 301

Lagrange multiplier, 270
Lagrangian coherent structure (LCS), 39, 243
as second derivative of FTLE, 260
exists without FTLE, 260
maximizes repulsion rate, 259
weakly, 261
Lagrangian derivative, 66
Lagrangian patterns, 243
Lagrangian perspective, 2
lambda lemma, 243
laminar flow, 228
Langevin equation, 193
Laplacian matrix, 114, 117, 123
eigenvalue spectrum of, 138
largest Lyapunov exponent, 246
LCE, *see* Lyapunov characteristic exponent
LCS, *see* Lagrangian coherent structure
leading eigenvalue, 145
leakage, 78, 84, 89, 119, 128
leaky bucket problem, 30
leaky tent map, 141
Lebesgue absolutely continuous invariant measure, 297
Lebesgue measure, 6
Lebesgue measure zero, 54, 60
Lebesgue singular function, 14
left eigenvalue/eigenvector, 85
left eigenvector, 63
left singular vector, 126
Lempel–Ziv method, 269
level-set in the thresholding, 115
lexicographic order, 183
lifetime escape function, 229, 230, 232
of a Henon map, 229
lifetime escape set, 230
of a tent map, 232
lifetime landscape function, 228
likelihood of character occurrences, 267
limit points, 180
linear dynamical systen, 8
linearized system, 32
Lipschitz continuity, 19
Lipschitz property, 19
lobe, 213
lobe dynamics, 149, 210, 217, 218, 242, 251
local unstable manifold, 37
logistic map, 2, 286
long time scales, 57
Lorenz attractor
as information process, 277
symbolic dynamics, 180
Lorenz butterfly, 153
attractor, 154
Lorenz map, 152
Lorenz system, 152
Lorenz's successive maxima map, 154
loss of correlation, 149
loss of precision, 152, 164
lossless compression, 268
lossy compression, 268
Lyapunov characteristic exponent (LCE), 239
positive, 240
Lyapunov exponent, 7, 168, 237, 239, 241, 297, 331
as stretching rates, 169
definition, 238
for diffeomorphism, 239
in the Gulf of Mexico, 28
logistic map, 238
Lyapunov number, 246
Lyapunov vector, 246

manifold
global stable, 38
global unstable, 38
local stable, 37, 40
local unstable, 37, 40
many-branched preimage, 283
marine ecosystem, 23
Markov chain, 2, 81, 91, 107
confidence-based description of, 194

represented by a transition matrix, 114
reversible, 316
Markov chaotic functions, 98
Markov map
dense, 95
density of, 100
Markov matrix
on a finite-dimensional vector space, 68
Markov model, 23, 61, 63, 190, 213, 217
imbedded, 291
of the attractor, 286
Markov partition, 3, 69, 73, 187, 286, 289, 309
for cat map, 75
Markov partition property, 177
Markov property
deviation from, 299
Markov transformation, 72, 73
piecewise linear, 98
uniformly convergent, 99
Markov tree model, 220
masked transfer operators, 225
mass conservation, 8
material derivative, 66
material manifold of a hyperbolic trajectory, 258
material surface, 243, 258
mathematical epidemiology, 224
MATLAB code, 309
matrix 1-norm, 91
matrix geometric series, 223
matrix infinity-norm, 91
matrix natural norm, 91, 223
matrix norm, 104
max norm, 87
maxima map
one-dimensional successive, 154
maximal Lyapunov exponent, 242
maximally almost-invariant, 115
maximizing entropy, 284
maximum entropy measure, 279
maximum stretching, 244
mean value theorem, 47
measurable dynamics, 29, 149, 197
measurable function, 49
measurable properties, 17, 29
measurable sets, 6
measurable space, 49
measure leaking, 143
measure preserving transformations, 44
measure space, 9, 49
nonsingular, 59
measure zero unstable chaotic saddle, 175
measurement precision, 150
Melnikov analysis, 175, 176
generalizations of, 179
Melnikov function, 176
Melnikov method, 169, 220
Melnikov theorem, 178
MET, *see* multiplicative ergodic theorem
metric entropy, 14, 282
metric-like distance, 274
metric space, 167
metric topology
definition, 285
metrizable compact space, 59
min-cut problem, 138
minimal transport, 119
minimum code word length, 275
minimum cut method, 138
misplaced partition, 152, 190
misplacement parameter, 190
modularity matrix, 139
monodromy matrix, 34
monotone nondecreasing, 14
Monte Carlo estimate
Ulam's method, 325
Monte Carlo integration, 324
Monte Carlo integration formula, 325
Monte Carlo method, 324
Monte Carlo simulations, 62
multiplicative ergodic theorem (MET)
geometric interpretation, 241
multiply branched
preimage, 52
multistable dynamical system, 83
mutual information, 273–275, 298, 305

N-cylinder, 163
n-step path, 81
natural barrier, 200

natural invariant measure, 54, 281
natural measure, 62, 63
    approximation of, 63
nested homoclinic and heteroclinic tangles, 216
noise-induced basin hopping, 224
non-information-bearing bits, 277
non-Markov case, 71
non-Markov partition, 182
nonautonomous differential equation, 16
nonautonomous equation, 16
nonautonomous vector field, 23
nongenerating partition, 192
nonidentical synchronization, 304
noninvertibility, 160
nonlinear dynamics, 286
nonmaximal topological entropy, 153
nonnegative matrix, 89, 90
nonoverlapping code, 268
nonsingular code, 269
nonsingular measures, 59
nonuniform grid, 310
normalized Laplacian, 119

objective function graph, 137
oceanic systems, 23
oceanographic processes, 308
ODE case, 31
ODE theory, 19
oil spill, 23
one-hump map, 183
open relative to the domain, 147
open system, 140, 223
optimal 2-partition, 115, 120
optimal coherent structure, 120
optimal instantaneous code, 271
optimal invariance, 115
optimal stochastic partition, 194
orbits, 4
    finite samples of, 180
ordering of basis functions, 104
orientable manifolds, 197
orientation of surface, 203
orientation-preserving map, 197
orthogonal projection, 88
orthogonality, 116
orthonormal eigenbasis, 244
orthonormal set of initial conditions, 34
Oseledets multiplicative ergodic theorem, 238
Oseledets multiplicative ergodic theorem (MET), 240
out-degrees, 135

p.i.p. segments, 218
parameterized curves, 151
parametric noise, 51
Parry's maximal entropy measure, 284
partial barrier, 220, 243
partial differential equation (PDE), 30, 47
partition of arbitrary rectangles, 188
partitioning a network, 133, 139
partitioning the phase space, 101
partitions of a graph into subgraphs, 80
passive tracers, 121
path, 81
PDE, *see* partial differential equation
PDE theory, 47
PDF, *see* probability density function
period three implies chaos, 183
periodic orbits
    uncountable, chaotic, infinitely many, 163
periodically forcing double gyre, 249, 251
permutation, 80, 107
permutation matrix, 82
Pesin entropy identity, 297
Pesin identity, 296
phase portrait, 16, 17
phase space, 17
phase space partitioning, 132
phase space transport, 51
Picard–Lindelöf theorem, 19
piecewise constant invariant density, 296
piecewise constant invariant PDF, 98
piecewise continuously differentiable map, 237
piecewise linear transformation, 96
piecewise linear, interval map, 98
piecewise linear, unimodal map, 94
pipe flow, 227
plane Couette flow, 227

Poincaré, H., 215
Poincaré map, 16
Poincaré mapping, 17, 20
    from the Duffing equations, 209
    stroboscopic method of, 176
Poincaré section, 17
Poincaré-stroboscopic mapping method, 21
Poincaré surface mapping, 152
Poincaré's trick, 18
pointwise convergence, 94
polar vortex, 120
positive Lyapunov exponent, 163, 169
positive matrix, 89
positively invariant, 54
potential vorticity, 132, 256
power method, 87
    geometric convergence of, 89
precompact set, 99
prefix code, 270
primitive, 81
primitive matrix, 81, 93
principle intersection point, 203, 204
probabilistic model, 267
probability density function, 272
probability density function (PDF), 44, 94
probability model, 266
product probabilities space, 271
projected transition matrix, 295
projection operator, 39
pruning theory, 174, 189
pseudobarriers, 23, 218
pseudo-orbit, 229
pseudorandom number, 8

quadratic cost function, 116
quadratic shear, 157
quadweb diagram, 13
quasi–Monte Carlo integration, 324
quasi-periodic system, 127
quasi-stationary distribution, 145

Radon–Nikodým derivative, 6
rain gauge measure, 62
random configuration, 106
random experiment, 49
random symbol, 281
random variable, 49, 91, 271, 281
    experimental outcome, 282
random walker, 143
rates of transport, 2
Rayleigh quotient, 114
reaction diffusion equations, 31
real symmetric matrix, 115
reducibility, 83
reducible graph, 82
reducible matrix, 93
relative coherent pair, 145
relative measure, 145
relatively coherent sets, 227
relaxation problem, 125
relaxation to the optimal generalized partition, 125
remaining entropy bits, 272
repelling Lagrangian coherent structure, 257
resonance, 212
resonance overlap, 217, 218
reverse time, 66
reversibility, 30
"reversibilizing" the transition matrix, 116
reversible Markov chain, 114, 138, 139
Riemannian differentiable manifold, 244
Riemannian manifold, 35
right eigenvalue, 85
right eigenvector, 85
right resolvent, 181
right resolvent shift, 182
right resolvent sofic shift, 285
right singular vector, 126
robust to small perturbation, 121
role of partitions, 150
Rossby system, 227
Rossby wave system, 147
Rossby waves, 127, 132
row-stochastic matrix, 89
row-wise stochastic, 86
rubber sheet geometry, 156

sampling method, 309
Šarkovs'kiĭ's theorem, 183
second eigenvector, 110, 316
second-derivative ridge of the FTLE field, 257

second-largest singular vector, 126
SEIR model, 225
self-similarity, 166
semiconjugate
  definition, 291
semidynamical system, 30, 32, 43
  invariant set of, 54
  nonsingular, 59
semiflow, 30, 31
semigroup, 30
semigroup action, 30
semigroup of contracting operators, 45
semigroup of transformations, 44
sensitive dependence, 152, 180
sensitive dependence on initial conditions, 167, 168, 284
separate basins, 226
separatrix, 249, 251
sequence of random variables, 281
sequences of a chaotic orbit, 191
set oriented method, 71
Shannon entropy, 265, 268–270
Shannon information, 270, 271
Shannon–Fano code, 269
Shannon–McMillan–Breiman theorem, 282
shift map, 60
short time scale, 24
shortest path, 133
shuffling cards, 169
sigma algebra, 6
sign structure, 120
signed area, 203
signs of the second eigenvector, 110, 143
similarity transformation, 82, 107
Sinai–Ruelle–Bowen measure, 288
singular spectrum, 90
singular vector, 246
singularly perturbed system, 56
skew tent map, 94
slow manifold, 56, 57
slow transport between almost invariant sets, 101
Smale, 207
Smale homoclinic theorem, 152
Smale horseshoe, 38
Smale horseshoe theorem, 207
Smale–Birkhoff homoclinic theorem, 38
smooth topological manifold, 157
social connections, 133
sofic shift, 180, 182, 295
sojourn time, 215
Sort function, 110
source coding, 271
space averages, 60
sparse graph, 133
spatiotemporal data, 306
spatiotemporal system, 274
spectral circle, 90
spectral method, 138
spectral radius, 87, 91
spectrum of eigenvalues, 90
spectrum of the stochastic matrices, 89
square root of the matrix, 118
stable invariant set in a flow, 56
stable manifold theorem, 37, 201
stationary process, 283
statistical expectation, 268
statistical independence, 275
statistical model, 266
statistical physics, 60
stochastic differential equation, 49
stochastic dynamical system, 5
stochastic Frobenius–Perron operator, 193
stochastic layer, 255
stochastic matrices from Frobenius–Perron operator, 107
stochastic matrix, 2, 63, 78, 90, 107
stochastic matrix estimate, 54
stochastic matrix Ulam–Galerkin estimate of the Frobenius–Perron operator, 315
stochastic perturbation
  additive, 51
  additive and multiplicative, 51
  multiplicative, 51
stochastic process, 282
stochastically perturbed block, 111
stochastically perturbed with Gaussian noise, 194

stratified Monte Carlo estimator, 327
stratospheric flow, 132
stratospheric surf zone, 132
stretch+fold in the Duffing attractor, 176
stretch+fold process, 162, 171
stretched and folded, 49
stroboscopic map, 252
stroboscopic section, 18
strong community partition, 135
strong derivative, 46
subgroup construction, 183
submaximal entropy, 177
subshift, 155
  definition, 180
  embedded, 291, 292
  on two symbols, 190
  topological entropy of, 181, 190
  topologically a closed set, 180
substochastic matrix, 144
successive maxima map, 153
successive refinements, 321
SVD decomposition, 90
symbol dynamics, 12, 72, 73, 149
  symbol partition, 281
  topological perspective of, 193
symbol set, 156
symbolic dynamical descriptions, 206
symbolic dynamics, 71, 160, 197, 220, 276
symbolic false nearest neighbors, 194
symbolic metric topology, 164
symbolic sequence, 190
symbolic stream, 193
symmetric bilinear form, 33
symmetric graph, 114
symmetric linear map, 246

tangencies of stable and unstable manifolds, 76
tangency bifurcation, 177
tangent bundle, 34
tangent space, 35
tent map
  symbolic dynamics, 190
tessellation, 69
  grid as, 309
thermodynamics, 60
thermodynamics theory, 60
threshold crossing, 191
threshold-crossing method
  symbolic dynamics, 189
thresholding, 107
thresholding the FTLE field, 248
time averages, 60
time dependencies, 298
time of first escape, 223
time series, 16
time-dependent flow, 246
time-dependent Lagrangian coherent structure, 246
time-dependent stream function, 251
time-reversed chain, 116
topological dynamical systems, 58
topological dynamics, 29, 71, 149, 197
topological entropy, 96, 184, 190, 279, 281, 284, 285
  as number of fixed points, 286
topological entropy equivalence, 292
topological entropy of factors, 291
topological entropy word count, 287
topological horseshoe, 158, 174, 206, 207
topological horseshoe geometry, 173
topological partition, 73, 75, 200
topological properties, 17, 29
topological space, 73, 149
topological transitivity, 63, 180
topologically conjugate, 98
topologically equivalent, 13, 72
toral automorphism, 75
total derivative, 66
total entropy, 272
traffic flow, 133
transfer entropy, 298, 299, 301
  as conditional Kullback–Leibler divergence, 301
transfer operator, 1
  for topological entropy for Henon, 294
transient turbulence, 227
transition matrix, 81
  invariant measure of, 63
transition probabilities, 69
transitions with no surprise, 279

transitivity, 167
transmission rate, 276
transport, 214
  celestial mechanics, 218
  geometric description of, 218
transport across an arbitrary barrier, 204
transport analysis, 22
transport and mixing theory, 38
transport barrier, 253
transport between two basins, 84
transport mechanism, 197
  FTLE role, 246
transport on a graph, 101
transport rates, 218
transport study, 19
transport within window, 140
transverse heteroclinic intersection, 205
transverse homoclinic connection, 203
transverse homoclinic intersection, 205
transverse homoclinic point, 176, 178, 207, 208
transverse intersection, 178, 203
transverse intersection of stable and unstable manifolds, 209
transverse to the flow, 17
trellis structures, 220
troposphere, 132
true leading Lyapunov vector, 245
turnstile, 204
turnstiles
  stretching, 205
two-symbol partition, 154
two-to-one-ness, 160

u-change of variable, 14
Ulam conjecture, 15, 68
Ulam method, xiii, 22, 54, 63
  measurable dynamics perspective of, 193
  modification of, 122
  Monte Carlo integration, 324
Ulam, S., 68
Ulam–Galerkin matrices, 225
Ulam–Galerkin matrix, 70, 292, 309, 328
Ulam–Galerkin method, 22, 26, 67
Ulam–Galerkin representation, xi
uncountable state chain, 93
uniform limit, 94
uniformly continuously differentiability, 327
uniformly hyperbolic system, 35
unique ergodicity, 10, 60
uniqueness, 9
uniqueness of dominant states, 93
unitary matrix, 109
unstable Cantor set, 174
unstable chaotic saddle, 54, 141, 228, 232
  definition, 231
unstable manifold analysis, 19
unstable manifolds
  of hyperbolic fixed points, 242
unstable material line, 253
unstable space, 202
UPOs, 286
upper triangular, 82

Van der Pol oscillator, 156
variation of parameters, 57
variational equation, 33, 242
vector field, 25
  nonautonomous, 39
vertices, 133
viscous friction, 16

weak approximation, 68
weak limit, 94
  of a sequence of functions, 95
weakly transitive behavior, 101
weighted directed graph, 78
“wild”, 150

Zachary’s Karate Club, 133