Preface

This book provides a comprehensive treatment of a very successful class of methods for solving optimization problems with PDE and inequality constraints as well as variational inequalities in function spaces. The approach combines the idea of nonsmooth pointwise reformulations of systems of inequalities with the concept of semismooth Newton methods. The book originates from the author's Habilitation thesis, in which the by then intensively investigated semismooth approach for finite-dimensional complementarity problems and variational inequalities was extended to and investigated in a function space setting. It was not predictable in 2000 that ten years later semismooth Newton methods would be one of the most important approaches for solving inequality constrained optimization problems in function spaces. The book develops this theory in detail; discusses recent progress, such as results on mesh independence, state constraints, and L^1-optimization; and shows applications ranging from obstacle problems to flow control. It is the author's hope that this book will be helpful for the future development of this exciting field.

The success of the semismooth approach in PDE constrained optimization and related fields was preceded by exciting research on semismooth Newton methods in finite dimensions and their application to complementarity problems. Mifflin's (1977) notion of semismoothness and the first papers on semismooth Newton methods, authored by Qi (1993), Qi and Sun (1993), and Pang and Qi (1993) formed important pillars for these developments. On the infinite-dimensional side, several abstract concepts for nonsmooth Newton methods in Banach spaces had been developed, e.g., by Kummer (1988, 1992), Robinson (1994), and Chen, Nashed, and Qi (2000). For transferring the full power of nonsmooth Newton methods to the function space setting, it was, however, crucial to investigate superposition operators with nonsmooth outer function, which occur when pointwise complementarity systems are reformulated as nonsmooth equations. This step was first done by the author (2001, 2002) and by Hintermüller, Ito, and Kunisch (2002). The latter paper also contains the important observation that the primal dual active set strategy can be interpreted as a semismooth Newton method. Since then, many contributions have been made to the field and the research is ongoing. Due to space limitations, I had to make a selection of topics that are presented in this book. I think that this choice is attractive and well suited for enabling the reader to follow the ongoing research in the field. Particular features of this book are

- rigorous development of the theory of semismooth Newton methods in a function space setting.
- mesh-independence results for semismooth Newton methods.
- regularizations and their rate of convergence for problems with state constraints.

- a globalization strategy based on a trust region framework.
- applications to elliptic optimal control, obstacle, and instationary flow control problems.

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München, January 2011 Michael Ulbrich