Preface

This book emerged from the idea that an optimization training should include three basic components: a strong theoretical and algorithmic foundation, familiarity with various applications, and the ability to apply the theory and algorithms on actual "real-life" problems. The book is intended to be the basis of such an extensive training. The mathematical development of the main concepts in nonlinear optimization is done rigorously, where a special effort was made to keep the proofs as simple as possible. The results are presented gradually and accompanied with many illustrative examples. Since the aim is not to give an encyclopedic overview, the focus is on the most useful and important concepts. The theory is complemented by numerous discussions on applications from various scientific fields such as signal processing, economics and localization. Some basic algorithms are also presented and studied to provide some flavor of this important aspect of optimization. Many topics are demonstrated by MATLAB programs, and ideally, the interested reader will find satisfaction in the ability of actually solving problems on his or her own. The book contains several topics that, compared to other classical textbooks, are treated differently. The following are some examples of the less common issues.

- The treatment of stationarity is comprehensive and discusses this important notion in the presence of sparsity constraints.
- The concept of "hidden convexity" is discussed and illustrated in the context of the trust region subproblem.
- The MATLAB toolbox CVX is explored and used.
- The gradient mapping and its properties are studied and used in the analysis of the gradient projection method.
- Second order necessary optimality conditions are treated using a descent direction approach.
- Applications such as circle fitting, Chebyshev center, the Fermat–Weber problem, denoising, clustering, total least squares, and orthogonal regression are studied both theoretically and algorithmically, illustrating concepts such as duality. MATLAB programs are used to show how the theory can be implemented.

The book is intended for students and researchers with a basic background in advanced calculus and linear algebra, but no prior knowledge of optimization theory is assumed. The book contains more than 170 exercises, which can be used to deepen the understanding of the material. The MATLAB functions described throughout the book can be found at

`www.siam.org/books/mo19.`

The outline of the book is as follows. **Chapter 1** recalls some of the important concepts in linear algebra and calculus that are essential for the understanding of the mathematical developments in the book. **Chapter 2** focuses on local and global optimality conditions for smooth unconstrained problems. Quadratic functions are also introduced along with their basic properties. Linear and nonlinear least squares problems are introduced and studied in **Chapter 3**. Several applications such as data fitting, denoising, and circle fitting are discussed. The gradient method is introduced and studied in **Chapter 4**. The chapter also contains a discussion on descent direction methods and various stepsize strategies. Extensions such as the scaled gradient method and damped Gauss–Newton are considered. The connection between the gradient method and Weiszfeld's method for solving the Fermat–Weber problem is established. Newton's method is discussed in **Chapter 5**. Convex sets and functions along with their basic properties are the subjects of **Chapters 6 and 7**. Convex optimization problems are introduced in **Chapter 8**, which also includes a variety of applications as well as CVX demonstrations. **Chapter 9** focuses on several important topics related to optimization problems over convex sets: stationarity, gradient mappings, and the gradient projection method. The chapter ends with results on sparsity constrained problems, illuminating the different type of results obtained when the underlying set is not convex. The derivation of the KKT optimality conditions from the separation and alternative theorems is the subject of **Chapter 10**, where only linearly constrained problems are considered. The extension of the KKT conditions to problems with nonlinear constraints is discussed in **Chapter 11**, which also considers the second order necessary conditions. Applications of the conditions to the trust region and total least squares problems are studied. The book ends with a discussion on duality in **Chapter 12**. Strong duality under convexity assumptions is established. This chapter also includes a large amount of examples, applications, and MATLAB illustrations.

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