Preface

This monograph explores mathematical issues related to vulnerability analysis of power grids, with a focus on optimization methods. The target audience is primarily made up of operations researchers and applied mathematicians with an interest in electrical power transmission, as well as power engineering specialists who are interested in optimization topics, especially nonconvex optimization and optimization under uncertainty. This monograph will present outlines of appropriate optimization and power engineering fundamentals before embarking on the study of the main topic, which is the rigorous analysis of power grid security. The level of mathematics is intermediate, with occasional developments at greater depth and even some detailed proofs.

The focus of this monograph is the power grid, which can be considered one of the greatest achievements of twentieth-century engineering, and from which populations around the world have attained an improved standard of living. Moreover, modern grids operate around the clock with remarkable constancy and reliability, and the secure delivery of power is taken for granted. These facts can be further explored as follows. The operation of power grids is driven by two core goals: (i) efficient economics, and (ii) secure, i.e., stable, operation. The first goal naturally gives rise to extensive use of optimization at many time scales. The second goal requires a strict adherence to operational criteria designed to keep a grid within a narrowly defined set of available states. This is an overriding principle that trumps even the need for efficiency—it is constantly applied to an extent that a non-power specialist may find surprising. This principle is to a large extent framed through the use of systems of quadratic inequalities designed to reflect the underlying physics of power transport and generation. Often, such nonlinear and nonconvex feasibility problems are recast as optimization problems to account for missing information or infeasibility conditions.

The integration of efficiency and security implies, as a consequence, the need for accurate and efficient optimization as a basic ingredient of grid operation. Altogether the record has been very good. Grids are attaining increasing efficiency and have, overall, proved quite stable.

However, as grids and populations have grown, this success has been punctuated by some high-visibility failures. In fact, during the last fifteen years several significant blackouts have taken place around the world, affecting tens of millions of people and causing significant economic and social disruption. All of these events were caused by a combination of accidental, exogenous causes and human error. The potential for more serious events remains high. Recently, the U.S. National Academies issued a report warning of the possibility of malevolently caused blackouts, carrying an even higher risk.

Blackouts are often complex events because they are caused by a cascade of failures that accelerates and outstrips control capabilities. In a cascade, the disabling of a piece of equipment typically causes other pieces of equipment to become overloaded, which in turn increases the likelihood of their failure. Should this process become self-sustaining it
will tend to accelerate, overwhelming control capabilities and finally causing a large loss of power delivery—a national-scale blackout.

Since blackouts do happen, it is important to develop methodology for understanding the mechanism of an ongoing cascading failure so that, for example, one can develop fast control algorithms that are provably good, that is to say provably fast and effective. This goal requires appropriate modeling of myriad issues, such as an explanation of how equipment fails and how electrical power “load” is redistributed following a set of outages. As we will discuss, this is an ambitious goal because of the nature of the physics that underlies electrical power, the complexity of the equipment and existing control schemes, and the overabundance of numerical parameters that ideally should be precisely calibrated but in practice are sometimes unknown. A broad part of this monograph targets these issues.

A separate issue concerns the prevention of cascading failures, that is to say the goal of achieving “stability,” broadly defined. Modern power grids are clearly stable in the sense that they rarely fail. But since they do occasionally fail, an important question is what structural features constitute a weakness that would be revealed in a failure. The search for such hidden weaknesses becomes an important planning task. The word “hidden” is appropriate in this context since those weaknesses that do remain, despite ongoing improvement efforts, can be considered complex or counterintuitive. The development of an appropriate agnostic methodology for rooting out such weaknesses is therefore an important task. This problem, which is also framed as an optimization problem, is frequently known as the N-K problem and constitutes a second focus of this work.

The need to resolve the above modeling questions in an agnostic manner gives rise to a host of complex and challenging optimization problems, incorporating many facets of modern optimization theory and practice, including linear and nonlinear programming; convex, nonconvex, and discrete optimization; and stochastic and robust optimization. As noted above, this is a natural rather than a forced modeling outcome.

This monograph is organized as follows. Chapter 1 provides an introduction to power engineering which should prove accessible to a reader with a basic applied mathematics background, as well as mathematical statements of several problems addressed in power grid operations. Chapter 2 focuses on mathematical topics related to the computation of power flows in a grid, i.e., the basic computation underlying grid operations today, and introduces the use of semidefinite relaxations, filter methods, and derivative-free methods in this context. Chapter 3 considers the N-K problem, the basic computational task used to assess the reliance of a grid, and describes the use of mixed-integer, nonconvex, and chance-constrained optimization as applied to the N-K problem. Chapter 4 introduces and surveys the modeling of cascading failures of transmission systems, followed by more detailed modeling in Chapter 5. Chapter 6 describes some initial work on rigorous online control schemes that could be used to stop, or slow down, a cascading failure, as well as more detailed models of cascading failures, and appropriate mathematical optimization methods. Throughout, we rely on extensive use of examples designed to reveal counterintuitive behavior of power systems, especially issues related to nonmonotonicity, nonconvexity, and Braess’s paradox. Data not given in the text is provided on the book’s webpage, www.siam.org/books/mo22.