

Contents

List of Figures	ix
List of Algorithms	xi
Preface	xiii
1 Introduction	1
1.1 Why ellipsoids?	1
1.2 The minimum-volume enclosing ellipsoid problem	4
1.3 Optimal design in statistics	6
1.4 Applications	7
1.5 Outline of the book	9
1.6 Notes and references	9
2 Minimum-Volume Ellipsoids	11
2.1 Duality, existence, and uniqueness	11
2.2 Optimality conditions	16
2.3 Relaxing the centered restriction	17
2.4 Quality of fit of minimum-volume enclosing ellipsoids	20
2.5 Notes and references	22
3 Algorithms for the MVEE Problem	25
3.1 Coordinate-ascent algorithms	26
3.2 Initialization	30
3.3 Global convergence and complexity	33
3.4 Local convergence	38
3.5 Polarity and a striking relationship to the ellipsoid algorithm	40
3.6 Small core sets and eliminating points	42
3.7 A connection to spectral sparsification of graphs	44
3.8 Computational results	46
3.9 Notes and references	49
4 Minimum-Area Ellipsoidal Cylinders	51
4.1 Formulations of the MAEC problem	51
4.2 Duality for the MAEC problem	54
4.3 Optimality conditions for the MAEC problem	60
4.4 D_k -optimal design in statistics	63
4.5 Collision detection	64
4.6 Notes and references	65

5	Algorithms for the MAEC Problem	67
5.1	Derivative properties of the dual objective function	68
5.2	Coordinate-ascent algorithms	71
5.3	Global convergence	75
5.4	Local convergence	79
5.5	Rank deficiency	82
5.6	Computational results	85
5.7	Notes and references	86
6	Related Problems and Algorithms	89
6.1	Conditional minimum-volume ellipsoids	89
6.2	Approximating by parallelotopes	95
6.3	Maximum-volume ellipsoids inscribed in a polyhedron	97
6.4	Notes and references	106
A	Background Material	109
A.1	Notation, inner products, and norms	109
A.2	Positive (semi)definiteness	110
A.3	Schur complements and low-rank updates	115
A.4	Matrix analysis	117
A.5	Convexity	118
A.6	Optimality conditions and duality	119
A.7	Compactness of the set of direction matrices	121
A.8	Derivation of a dual to the maximum-volume inscribed ellipsoid problem	122
B	MATLAB Codes	125
	Bibliography	143
	Index	149