

6.9 Summary of Prox Computations

$f(\mathbf{x})$	$\text{dom}(f)$	$\text{prox}_f(\mathbf{x})$	Assumptions	Reference
$\frac{1}{2}\mathbf{x}^T \mathbf{A} \mathbf{x} + \mathbf{b}^T \mathbf{x} + c$	\mathbb{R}^n	$(\mathbf{A} + \mathbf{I})^{-1}(\mathbf{x} - \mathbf{b})$	$\mathbf{A} \in \mathbb{S}_+^n$, $\mathbf{b} \in \mathbb{R}^n$, $c \in \mathbb{R}$	Section 6.2.3
λx^3	\mathbb{R}_+	$\frac{-1 + \sqrt{1 + 12\lambda x }}{6\lambda}$	$\lambda > 0$	Lemma 6.5
μx	$[0, \alpha] \cap \mathbb{R}$	$\min\{\max\{x - \mu, 0\}, \alpha\}$	$\mu \in \mathbb{R}$, $\alpha \in [0, \infty]$	Example 6.14
$\lambda \ \mathbf{x}\ $	\mathbb{E}	$\left(1 - \frac{\lambda}{\max\{\ \mathbf{x}\ , \lambda\}}\right) \mathbf{x}$	$\ \cdot\ $ —Euclidean norm, $\lambda > 0$	Example 6.19
$-\lambda \ \mathbf{x}\ $	\mathbb{E}	$\begin{cases} \left(1 + \frac{\lambda}{\ \mathbf{x}\ }\right) \mathbf{x}, & \mathbf{x} \neq \mathbf{0}, \\ \{\mathbf{u} : \ \mathbf{u}\ = \lambda\}, & \mathbf{x} = \mathbf{0}. \end{cases}$	$\ \cdot\ $ —Euclidean norm, $\lambda > 0$	Example 6.21
$\lambda \ \mathbf{x}\ _1$	\mathbb{R}^n	$\mathcal{T}_\lambda(\mathbf{x}) = [\mathbf{x} - \lambda \mathbf{e}]_+ \odot \text{sgn}(\mathbf{x})$	$\lambda > 0$	Example 6.8
$\ \boldsymbol{\omega} \odot \mathbf{x}\ _1$	$\text{Box}[-\boldsymbol{\alpha}, \boldsymbol{\alpha}]$	$S_{\boldsymbol{\omega}, \boldsymbol{\alpha}}(\mathbf{x})$	$\boldsymbol{\alpha} \in [0, \infty]^n$, $\boldsymbol{\omega} \in \mathbb{R}_+^n$	Example 6.23
$\lambda \ \mathbf{x}\ _\infty$	\mathbb{R}^n	$\mathbf{x} - \lambda P_{B_{\ \cdot\ _\infty}[0,1]}(\mathbf{x}/\lambda)$	$\lambda > 0$	Example 6.48
$\lambda \ \mathbf{x}\ _a$	\mathbb{E}	$\mathbf{x} - \lambda P_{B_{\ \cdot\ _a}[0,1]}(\mathbf{x}/\lambda)$	$\ \cdot\ _a$ —arbitrary norm, $\lambda > 0$	Example 6.47
$\lambda \ \mathbf{x}\ _0$	\mathbb{R}^n	$\mathcal{H}_{\sqrt{2\lambda}}(x_1) \times \cdots \times \mathcal{H}_{\sqrt{2\lambda}}(x_n)$	$\lambda > 0$	Example 6.10
$\lambda \ \mathbf{x}\ ^3$	\mathbb{E}	$\frac{2}{1 + \sqrt{1 + 12\lambda\ \mathbf{x}\ }} \mathbf{x}$	$\ \cdot\ $ —Euclidean norm, $\lambda > 0$,	Example 6.20
$-\lambda \sum_{j=1}^n \log x_j$	\mathbb{R}_{++}^n	$\left(\frac{x_j + \sqrt{x_j^2 + 4\lambda}}{2}\right)_{j=1}^n$	$\lambda > 0$	Example 6.9
$\delta_C(\mathbf{x})$	\mathbb{E}	$P_C(\mathbf{x})$	$\emptyset \neq C \subseteq \mathbb{E}$	Theorem 6.24
$\lambda \sigma_C(\mathbf{x})$	\mathbb{E}	$\mathbf{x} - \lambda P_C(\mathbf{x}/\lambda)$	$\lambda > 0$, $C \neq \emptyset$ closed convex	Theorem 6.46
$\lambda \max\{x_i\}$	\mathbb{R}^n	$\mathbf{x} - \lambda P_{\Delta_n}(\mathbf{x}/\lambda)$	$\lambda > 0$	Example 6.49
$\lambda \sum_{i=1}^k x_{[i]}$	\mathbb{R}^n	$\mathbf{x} - \lambda P_C(\mathbf{x}/\lambda)$, $C = H_{\mathbf{e}, k} \cap \text{Box}[\mathbf{0}, \mathbf{e}]$	$\lambda > 0$	Example 6.50
$\lambda \sum_{i=1}^k x_{[i]} $	\mathbb{R}^n	$\mathbf{x} - \lambda P_C(\mathbf{x}/\lambda)$, $C = B_{\ \cdot\ _1}[\mathbf{0}, k] \cap \text{Box}[-\mathbf{e}, \mathbf{e}]$	$\lambda > 0$	Example 6.51
$\lambda M_f^\mu(\mathbf{x})$	\mathbb{E}	$\mathbf{x} + \frac{\lambda}{\mu + \lambda} (\text{prox}_{(\mu + \lambda)f}(\mathbf{x}) - \mathbf{x})$	$\lambda, \mu > 0$, f proper closed convex	Corollary 6.64
$\lambda d_C(\mathbf{x})$	\mathbb{E}	$\mathbf{x} + \min\left\{\frac{\lambda}{d_C(\mathbf{x})}, 1\right\} (P_C(\mathbf{x}) - \mathbf{x})$	$\emptyset \neq C$ closed convex, $\lambda > 0$	Lemma 6.43
$\frac{\lambda}{2} d_C^2(\mathbf{x})$	\mathbb{E}	$\frac{\lambda}{\lambda + 1} P_C(\mathbf{x}) + \frac{1}{\lambda + 1} \mathbf{x}$	$\emptyset \neq C$ closed convex, $\lambda > 0$	Example 6.65
$\lambda H_\mu(\mathbf{x})$	\mathbb{E}	$\left(1 - \frac{\lambda}{\max\{\ \mathbf{x}\ , \mu + \lambda\}}\right) \mathbf{x}$	$\lambda, \mu > 0$	Example 6.66
$\rho \ \mathbf{x}\ _1^2$	\mathbb{R}^n	$\begin{cases} \left(\frac{v_i x_i}{v_i + 2\rho}\right)_{i=1}^n, \mathbf{v} = \\ \left[\sqrt{\frac{\rho}{\mu}} \mathbf{x} - 2\rho\right]_+, \mathbf{e}^T \mathbf{v} = 1 \ (\mathbf{0} \\ \text{when } \mathbf{x} = \mathbf{0}) \end{cases}$	$\rho > 0$	Lemma 6.70
$\lambda \ \mathbf{A}\mathbf{x}\ _2$	\mathbb{R}^n	$\mathbf{x} - \mathbf{A}^T (\mathbf{A}\mathbf{A}^T + \alpha^* \mathbf{I})^{-1} \mathbf{A}\mathbf{x}$, $\alpha^* = 0$ if $\ \mathbf{v}_0\ _2 \leq \lambda$; otherwise, $\ \mathbf{v}_{\alpha^*}\ _2 = \lambda$; $\mathbf{v}_\alpha \equiv (\mathbf{A}\mathbf{A}^T + \alpha \mathbf{I})^{-1} \mathbf{A}\mathbf{x}$	$\mathbf{A} \in \mathbb{R}^{m \times n}$ with full row rank, $\lambda > 0$	Lemma 6.68