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Lectures on Stochastic Programming: Modeling and Theory. *By A. Shapiro, D. Dentcheva, and A. Ruszczyński.* SIAM, Philadelphia, 2009. $119.00. xvi+436 pp., softcover. MOS-SIAM Series on Optimization. Vol. 9. ISBN 978-0-898716-87-0.

Stochastic programming emerged as a new field with the pioneering paper [4] by Dantzig, whose main result was that, in the case of linear constraints with a random right-hand side and a convex cost function, the resulting problem was convex. Therefore, some of these problems could be solved using the technology of that time, either linear programming or the cutting plane algorithm [7]. Another early contribution was the paper [3] by Charnes and Cooper on chance-constrained programming, i.e., on optimization problems with constraints which are to be satisfied with a certain probability. It was soon recognized that the simple recourse problem (in which the realization of the uncertain variable is revealed in a single time) is much simpler than multiple recourse problems, in which a sequence of partial information and decisions occurs. One can find in the classical textbooks by Kall and Wallace [6] and Birge and Louveaux [2] an account of the numerical techniques which were developed based on the scenario tree approach, some techniques to deal with probability constraints, and quasi-gradient algorithms.

When expressing the optimality conditions of stochastic programming problems, one often has to compute the subdifferential of a convex function, which is an integral, leading to expressions that in applications typically involve the dual of L^∞ spaces. Such difficult questions were analyzed by (among many people) Rockafellar [8]. Other theoretical issues were discussed by Wets [10].

Still, until the end of the last century the standard approach in applications was to solve scenario tree approximations, without much guarantee of the quality of the numerical result. An essential advance has been the derivation of the statistical properties of the sampling approach (generation of events by sampling) based on deep functional central limit theorems [1]. Rigorous statistical estimates could be provided for such problems. However, it was found that multistage problems suffer from the curse of dimensionality.

The present book presents the basic concepts that should be well understood by anyone interested in this field. It may be divided into two parts. The three first chapters introduce general models and concepts. Chapter 1 discusses simple examples such as the news vendor problem, multiproduct assembly, portfolio selection, and supply chain, some of them including chance constraints. Chapter 2 presents problems with simple recourse, with some material on polyhedral problems, followed by the study of the general case and an analysis of nonanticipativity constraints. Chapter 3, devoted to multistage problems, introduces scenario trees for (general) random processes with a finite number of realizations. It presents optimality conditions and duality for the cost-to-go function (involved in the dynamic programming principle).

The next three chapters deal with advanced concepts. Chapter 4 is devoted to the study of probabilistic constraints. It discusses cases where the latter happen to be convex, the concept of a p-efficient point, duality and optimality conditions, the approximation of nonseparable probabilistic constraints, and semi-infinite probabilistic constraints. Chapter 5 analyzes the statistical properties of the sample average approximation estimators, which amounts to extending central theorems to the case of a minimization problem, which is possible, as mentioned before, thanks to functional central limit theorems combined with delta theorems. It provides exponential rates of convergence in the case of a finite feasible set. Quasi-Monte Carlo and variance reduction methods are presented. The Monte Carlo sampling approach to chance constrained problems is analyzed. The (bad) complexity estimate for multistage problems is obtained. Robust quasi-gradient algorithms are presented. Chapter 6 is devoted to a branch of stochastic programming that appeared and was extensively developed in the last decade, i.e., the inclusion of risk measures in the optimization model. The usual measures such as Var, CVaR, etc., are reviewed and the link to utility functions and stochastic dominance is discussed. Extension to multistage problems (involving so-called conditional risk mappings) leads to an extended dynamic programming principle in this setting. Let us mention finally Chapter 7, which collects some relevant mathematical material.

The book is published by SIAM in its series on optimization, coedited with the Mathematical Optimization Society (formerly the Mathematical Programming Society). Since it is devoted mainly to the presentation of the basic concepts, it might be useful to mention complementary material on software and applications in the book [9] of the same series, edited by Wallace and Ziemba, and material on risk measures in [5].

There has been so much progress in recent years in stochastic programming, especially concerning mathematical analysis, statistical estimates, and extended formulations (risk measures, dominance constraints), that the present book is a welcome answer to the need for renewed presentation of the theory. It will be very useful for practitioners and researchers in the field, as well as for Ph.D. students.

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