Preface

The main topic of this book is optimization problems involving uncertain parameters, for which stochastic models are available. Although many ways have been proposed to model uncertain quantities, stochastic models have proved their flexibility and usefulness in diverse areas of science. This is mainly due to solid mathematical foundations and theoretical richness of the theory of probability and stochastic processes, and to sound statistical techniques of using real data.

Optimization problems involving stochastic models occur in almost all areas of science and engineering, from telecommunication and medicine to finance. This stimulates interest in rigorous ways of formulating, analyzing, and solving such problems. Due to the presence of random parameters in the model, the theory combines concepts of the optimization theory, the theory of probability and statistics, and functional analysis. Moreover, in recent years the theory and methods of stochastic programming have undergone major advances. All these factors motivated us to present in an accessible and rigorous form contemporary models and ideas of stochastic programming. We hope that the book will encourage other researchers to apply stochastic programming models and to undertake further studies of this fascinating and rapidly developing area.

We do not try to provide a comprehensive presentation of all aspects of stochastic programming, but we rather concentrate on theoretical foundations and recent advances in selected areas. The book is organized into seven chapters. The first chapter addresses modeling issues. The basic concepts, such as recourse actions, chance (probabilistic) constraints, and the nonanticipativity principle, are introduced in the context of specific models. The discussion is aimed at providing motivation for the theoretical developments in the book, rather than practical recommendations.

Chapters 2 and 3 present detailed development of the theory of two-stage and multistage stochastic programming problems. We analyze properties of the models and develop optimality conditions and duality theory in a rather general setting. Our analysis covers general distributions of uncertain parameters and provides special results for discrete distributions, which are relevant for numerical methods. Due to specific properties of two- and multistage stochastic programming problems, we were able to derive many of these results without resorting to methods of functional analysis.

The basic assumption in the modeling and technical developments is that the probability distribution of the random data is not influenced by our actions (decisions). In some applications, this assumption could be unjustified. However, dependence of probability distribution on decisions typically destroys the convex structure of the optimization problems considered, and our analysis exploits convexity in a significant way.

Chapter 4 deals with chance (probabilistic) constraints, which appear naturally in many applications. The chapter presents the current state of the theory, focusing on the structure of the problems, optimality theory, and duality. We present generalized convexity of functions and measures, differentiability, and approximations of probability functions. Much attention is devoted to problems with separable chance constraints and problems with discrete distributions. We also analyze problems with first order stochastic dominance constraints, which can be viewed as problems with continuum of probabilistic constraints. Many of the presented results are relatively new and were not previously available in standard textbooks.

Chapter 5 is devoted to statistical inference in stochastic programming. The starting point of the analysis is that the probability distribution of the random data vector is approximated by an empirical probability measure. Consequently, the "true" (expected value) optimization problem is replaced by its sample average approximation (SAA). Origins of this statistical inference are in the classical theory of the maximum likelihood method routinely used in statistics. Our motivation and applications are somewhat different, because we aim at solving stochastic programming problems by Monte Carlo sampling techniques. That is, the sample is generated in the computer and its size is constrained only by the computational resources needed to solve the constructed SAA problem. One of the byproducts of this theory is the complexity analysis of two-stage and multistage stochastic programming. Already in the case of two-stage stochastic programming, the number of scenarios (discretization points) grows exponentially with an increase in the number of random parameters. Furthermore, for multistage problems, the computational complexity also grows exponentially with the increase of the number of stages.

In Chapter 6 we outline the modern theory of risk averse approaches to stochastic programming. We focus on the analysis of the models, optimality theory, and duality. Static and two-stage risk averse models are analyzed in much detail. We also outline a risk averse approach to multistage problems, using conditional risk mappings and the principle of "time consistency."

Chapter 7 contains formulations of technical results used in the other parts of the book. For some of these less-known results we give proofs, while others refer to the literature. The subject index can help the reader quickly find a required definition or formulation of a needed technical result.

Several important aspects of stochastic programming have been left out. We do not discuss numerical methods for solving stochastic programming problems, except in section 5.9, where the stochastic approximation method and its relation to complexity estimates are considered. Of course, numerical methods is an important topic which deserves careful analysis. This, however, is a vast and separate area which should be considered in a more general framework of modern optimization methods and to a large extent would lead outside the scope of this book.

We also decided not to include a thorough discussion of stochastic integer programming. The theory and methods of solving stochastic integer programming problems draw heavily from the theory of general integer programming. Their comprehensive presentation would entail discussion of many concepts and methods of this vast field, which would have little connection with the rest of the book.

At the beginning of each chapter, we indicate the authors who were primarily responsible for writing the material, but the book is the creation of all three of us, and we share equal responsibility for errors and inaccuracies that escaped our attention.

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