Preface

Eigenvalue problems are ubiquitous in engineering and science. This book presents a unified theoretical development of the two most important classes of algorithms for solving matrix eigenvalue problems: QR-like algorithms for dense problems, and Krylov subspace methods for sparse problems. I make no claim to completeness. My choice of topics reflects my own interests, a desire to keep the length of the book within reason, and a wish to complete the book within my lifetime.

Prerequisites

Readers of this book are expected to be familiar with the basic ideas of linear algebra and to have had some experience with matrix computations. The student who has absorbed a good chunk of my book *Fundamentals of Matrix Computations* [221] and developed a bit of mathematical maturity will be in a good position to appreciate this book. It is expected that the reader already knows the importance of eigenvalue computations.

How to read the book

Chapters 1 and 2 contain background material and were not meant to be read linearly. I suggest that you begin with Chapter 3 and just refer back to the earlier chapters as needed. Chapters 3, 4, 8, and 9 form the heart of the book. Perhaps I should include Chapter 6 on the generalized eigenvalue problem in this list as well. Read Chapter 5 only if you are interested in the details of the convergence theory. Read Chapter 7 if you have an urge to find out what is going on inside the bulge.

Even though I have instructed you to start with Chapter 3, I hope you will end up spending a good chunk of time in Chapter 2, which contains basic theoretical material on eigensystems. I invested a lot of effort in that chapter, and I believe that many readers, including those with very strong backgrounds, will learn new and interesting things there.

A substantial fraction of the book is embedded in the exercises, so please work as many of them as possible. I expect you to spend more time in the exercises than in the text proper. Many proofs of theorems are presented as exercises, which outline my ideas about how you could prove the results. I do not claim that these are the best possible proofs; undoubtedly some of them could be improved. I invite reader feedback.

More on the contents of the book

Chapter 3 introduces the tools we need for creating zeros in matrices. Then Chapter 4 introduces and discusses GR algorithms, including the QR algorithm, the differential qd algorithm, the LR, SR, and HR algorithms, each of which is useful for solving certain special types of eigenvalue problems. Chapter 6 discusses GZ algorithms, including the QZ algorithm, for the generalized eigenvalue problem. Chapter 8 on Product Eigenvalue Problems shows that the QZ algorithm for the generalized eigenvalue problem, the implicit QR algorithm for the singular value decomposition, and periodic QR algorithms for more general product eigenvalue problems, are all special cases (not generalizations) of the QR algorithm for the standard eigenvalue problem. Chapter 9 introduces Krylov subspace methods for large, sparse eigenvalue problems. The focus is on short Krylov runs with frequent implicit restarts. A generic Krylov process is presented first, and two methods for making implicit restarts are worked out. Then special cases, including the Arnoldi process and symmetric and unsymmetric Lanczos processes, are discussed. Special Krylov processes that preserve unitary, Hamiltonian, and symplectic structure, are also developed. Finally, product Krylov processes (meshing with Chapter 8) and block Krylov processes are considered.

Website

I have compiled a modest collection of MATLAB programs to go with the book. Some of the exercises refer to them, and you can download them from

www.siam.org/books/ot101

Acknowledgments

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