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Assignment Problems. *By Rainer Burkard, Mauro Dell'Amico, and Silvano Martello.* SIAM, Philadelphia, 2009. $110.00. xx+382 pp., hardcover. ISBN 978-0-898716-63-4.

This book is a comprehensive treatment of the various categories of assignment problems: linear sum, bottleneck, algebraic, balanced, quadratic, and multi-index. It begins with the origins of the discipline in the early 20th century, in its various guises, and progresses to modern results and an extensive bibliography totaling 669 references. This well-written book should be suitable as a graduate-level textbook, or as a reference or guide for the researcher not already heavily involved in the field.

If we have n women and n men and a known symmetric relationship of "friendship" between women and men, can we form n marriages where every marriage pairs a woman and a man who are already "friends"? Suppose now that rather than binary friendship, we have each woman and each man provide a total ordering of all the potential spouses. Can we form n marriages where, in any unmarried pair of a woman and man, either the man prefers his current wife, or the woman prefers her current husband? In other words, can we marry everybody so that there is no temptation to form a new marriage where both parties to the new marriage would prefer their new spouse? The latter is known as the "stable marriage problem." These types of matching problems are the most basic discussed in this book and it should not be too hard to imagine their application to other optimization problems that are not quite as fanciful.

After a precise and concise overview in the first chapter, the second chapter presents historical and mathematical discussions of several related areas of discrete mathematics. Combinatorial matrix theory is represented by permutation matrices through a theorem of Frobenius (1917) and by the assignment polytope through a theorem of Birkoff (1946). Graph theory is represented by the search for matchings in bipartite graphs through König's matching theorem (1916) and Hall's marriage theorem (1935). Combinatorial optimization is represented by flows in networks through the Ford–Fulkerson "max-flow equals min-cut" theorem (1956). With Chapter 2 also containing mentions of the work of a wide range of influential mathematicians, such as Cayley, Edmonds, Klee, and Brualdi, one gets a good sense of the history of this topic and its wide appeal.

Early chapters concentrate on which of the various theorems are equivalent and which are generalizations of others, along with their relationship to assignment problems. Here, as throughout the text, proofs are given carefully and completely, a very welcome feature. These careful proofs are a big part of why this should be an excellent text for a graduate student who already has some experience with the techniques of discrete mathematics (such as a previous course in graph theory or combinatorial optimization). Where the text strays from the central topics, careful references are given, such as several early references to Schrijver [1, 2].

Later chapters concentrate on more recent developments by formulating variations on the basic linear sum assignment problem. The goal is to present the state of the art in current research and the text always indicates where further research is most desirable. There is a consistent effort throughout to provide applications of the problems described. For a listing of the chapters, including subsection titles, see the complete table of contents posted on the book's website.

Speaking of the book's website, at www.assignmentproblems.com there is a collection of some related software. The "Didactic Software" are collections of Java applets which allow a reader to step through an algorithm and clearly see its results, for example, by highlighting edges or vertices of a graph. They seem very nicely done and useful for learning the material. Other software is raw code, ready to compile, in languages like Fortran or C++, from a variety of sources. If there is any explicit licensing of this software, the reviewer was unable to locate it even after examining a few source code files. While

the coverage and age of these routines are variable, a student or instructor may still find them useful. Incorporated in a mainstream open source package such as Sage, they could be even more useful. In addition to selective coverage with source code, the book includes excellent and detailed pseudocode for the principal algorithms.

The website also includes a (very short) list of errors and, along with the software, PDF files taken straight from the book containing a few pages introducing the type of problem the software solves. For the undecided purchaser, these are good samples of the book's content and style.

This is an excellent book for graduate study (including preparation for thesis work). It should be useful to researchers and practitioners who can use the book to ascertain known results and follow references to the literature as needed. The provided software and pseudocode should be useful for teaching and learning, and perhaps for production work. It should be part of any library that supports a graduate mathematics program or industrial applications of discrete optimization.

REFERENCES

[1] A. Schrijver, *Combinatorial Optimization: Polyhedra and Efficiency*, Springer, Berlin, 2003.

[2] A. Schrijver, *On the history of combinatorial optimization (till* 1960*)*, in Handbook of Discrete Optimization, K. Aardal, G. L. Nemhauser, and R. Weismantel, eds., Elsevier, Amsterdam, 2005, pp. 1–68.

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