Preface to the Second Edition

Since the publication of *Symmetry in Chaos* in 1992, substantial progress has been made on the mathematics and science underlying symmetric chaos. For example, the concept of *patterns on average* is based on the mathematics behind our symmetric images. Bruce Gluckman, Philippe Marcq, Josh Bridger, and Jerry Gollub conducted elegant experiments at Haverford College which show that patterns on average occur in the Faraday experiment—a classical experiment from fluid dynamics. We have described this experiment in the revised introductory chapter. On the mathematical side, attractor symmetries have been classified and methods for numerically determining the symmetry of higher dimensional analogues of our images have been developed. The mathematical results have been obtained in collaboration with Pete Ashwin, Ernie Barany, Michael Dellnitz, Ian Melbourne, and Matt Nicol.

In another direction, the increasing power of desktop computers has enabled significant improvement of the resolution of the images as well as improvements in the coloring algorithms. Although, with few exceptions, we have kept the computer-generated images shown in the first edition, we have tried to improve both the quality and coloring of the images. We have added one or two new pictures, mainly in Chapters 4 and 5, which we hope will make some of the mathematical explanations of chaos easier to read.

Aside from the changes mentioned above, we replaced a few of the nonmathematical images. We have also worked to improve some of the mathematical explanations and have made minor improvements throughout the text. We removed the appendix on Basic programs—since Basic is no longer a readily available or widely used computer language.

We have been delighted that many of our images have found their way into the mathematics community: The *Notices of the American Mathematical Society* and a number of mathematics and science textbooks have used several of our pictures as cover images, the *Joint Policy Board for Mathematics* featured symmetric chaos on its 1997 *Mathematics Awareness Week* poster, and the *Institute for Mathematics and its Applications* uses a symmetric chaos image as its logo (see Figure 5.13 (top)).

The intertwining of mathematics and art has increased greatly over the past fifteen years. Indeed, following the 'Art-Math' conferences started by Nat Friedman in 1992, there are now many regular interdisciplinary conferences, such as *Bridges*, which explore connections between art, architecture, mathematics, and the sciences. We have greatly enjoyed the privilege of being involved in several of these meetings. Using ideas from symmetric chaos, one of us (MF) has given several courses to junior and senior students in the art department at the University of Houston as well as leading seminars for local teachers in the Houston Teachers Institute. We would like to thank the University of Houston for their encouragement and support of our exploration of the interface between mathematics and art and its potential for impacting education.

Some further acknowledgments are in order. We have received grant support from the National Science Foundation (NSF) and the Texas Advanced Research Program for the mathematical research referred to above. In particular, while preparing this revision we received partial support from NSF grants DMS-0600927, DMS-0806321 (MF), and DMS-0604429 (MG).

Last, but not least, we would like to thank SIAM for publishing the second edition of *Symmetry in Chaos* and, in particular, Elizabeth Greenspan for her enthusiastic editorial assistance.

Houston and Columbus
January, 2009

Mike Field
Marty Golubitsky

Preface to the First Edition

In our mathematics research, we study how symmetry and dynamics coexist. This study has led to the pictures of symmetric chaos that we present throughout this book. Indeed, we have two purposes in writing this book: to present these pictures and to present the ideas of symmetry and chaos—as they are used by mathematicians—that are needed to understand how these pictures are formed. As you will see, the images of symmetric chaos are simultaneously complex and familiar; the complexity stems from the chaotic dynamics by which the pictures are produced, while the familiarity is due to the symmetry.

Although symmetry has long played a key role in mathematics and indeed in virtually all models of the universe, the study of chaotic dynamics in mathematics and its use in modeling physical phenomena is a more recent endeavor. It is worth noting that both words *symmetry* and *chaos* have standard meanings in the English language as well as technical definitions in mathematics. There are clear similarities between the everyday usage and the technical definitions for each of these words—but the similarity is rather more for symmetry and rather less for chaos. In both everyday usage and mathematics, symmetry has the sense of repetition. For example, symmetry gives unity to designs from the rose windows of great cathedrals to the wallpaper in your own home by repeating one design a large number of times.

Chaos, on the other hand, means 'without form'—the great void. Viewed in this light, it is difficult to see how chaos can be the subject of scientific inquiry—which is based on finding form and regularity in the physical world. In recent years, the term chaos has (perhaps unfortunately) been adopted by mathematicians and scientists to describe situations which exhibit the twin properties of complexity and unpredictability. Archetypal examples are the weather and the stock market—although complex and unpredictable, these examples are far from being without form or structure.

One of our goals for this book is to present the pictures of symmetric chaos—in part because we find them beautiful and in part because we have enjoyed showing them to our friends and think they may appeal to others. But we also want to present the ideas that are needed to produce these pictures. For although the methods by which these computer-generated images are obtained are relatively simple, it is difficult to conceive how they might have been discovered without an appreciation of the underlying mathematics on which they are based.

In the first two chapters we discuss in general how the pictures are produced and how they are related to the mathematical ideas of symmetry and chaos. The third chapter has the role of an intermezzo. As we noted previously, the images of symmetric chaos often seem quite familiar. So in Chapter 3 we have paired off a number of pictures from nature (diatoms, shellfish, flowers, etc.) and a number of decorative designs (from rose windows to corporate logos to ceramic tiles) with designs produced on the computer using symmetric chaos methods. The fourth chapter is devoted to a more detailed discussion of the simplest forms of chaotic dynamics.

As you will see, we use three mathematically different methods for computing our images. Indeed these images which we call *symmetric icons*, *quilts*, and *symmetric fractals* are quite different in character. The methods for producing the icons, quilts, and symmetric fractals are explained in detail in the last three chapters. Since many of the readers of this book may be familiar with fractal art—say as appears in the books *The Beauty of Fractals* by Heinz-Otto Peitgen and Peter Richter and *Fractals Everywhere* by Michael Barnsley—it is worth noting that the images we present have a different character from those found in fractal art. While fractal pictures have the sense of *avant garde* abstract modernism or surrealism, ours typically have the feel of classical designs.

In the first appendix we present the exact parameter values that we have used to produce the pictures of symmetric chaos found in this volume. In Appendix B we give detailed computer programs (written in QuickBasic) that will enable the reader to experiment on a home computer with the formulas for symmetric chaos presented in Chapters 5–7. The actual derivation of these formulas is found in the last two appendices—one for the icons and one for the quilts. These sections contain more technical mathematics than the previous chapters.

Though this book is concerned with symmetry and chaos and their relationship with pattern formation and geometric design and art, we have not attempted here to describe in depth the many possible threads that lead from this work, in part, because these issues have been discussed elsewhere. Indeed, many authors have written about symmetry and art, but perhaps none more elegantly than Hermann Weyl in his classic book *Symmetry*. There have also been a number of books on chaos—our favorites being Ian Stewart's *Does God Play Dice?* and James Gleick's *Chaos: Making a New Science*. Finally, the mathematical and scientific discipline of pattern formation, which underlies our own work, is discussed in *Fearful Symmetry: Is God a Geometer?*

There are a number of individuals whose help we wish to acknowledge. This help has taken a variety of forms, from deriving the basic theory on which our pictures of symmetric chaos are based, to helping with the computer programming needed to produce high resolution color graphics on workstations, to making helpful suggestions on how better to use color in our pictures. We thank Peter Ashwin, Pascal Chossat, Robert Cottingham, Michael Dellnitz, April Field, Nathan Field, Michael Flanagan, Elizabeth Golubitsky, Phil Holmes, Barbara Keyfitz, Greg King, Martin Krupa, Ian Melbourne, Ralph Metcalfe, Jim Richardson, Harry Swinney, Hans True, and, in particular, Ian Stewart. We also thank Wendy Aldwyn, who has produced the hand-drawn artwork, including several original drawings, and has helped us with the coloring of several of the computer drawn pictures.

Layout, mathematical notations, graphics and color reproduction have made this book unusually complex to produce. Special thanks are due to Oxford University Press for their comprehensive assistance and helpful suggestions at every stage of the production process. The computer generated pictures were produced directly from computer files by Kaveh Bazargan of Focal Image, Ltd. using state-of-the-art processes. Finally, we acknowledge the institutional assistance of the Mathematical Sciences Institute, Cornell University; the Department of Mathematics, University of Houston; and the Department of Pure Mathematics, University of Sydney, for providing the intellectual and computer environments needed to produce these pictures.

Sydney and Houston | Mike Field
February 1992 | Marty Golubitsky