

# Index

(Page numbers set in **bold** type indicate the definition of an entry.)

- A**
- absolute error . . . . . **26**
    - componentwise . . . . . 31
    - in subtraction . . . . . 27
    - normwise . . . . . 31
  - angle in least squares
    - problem . . . . . 98, 99
  - approximation to 0 . . . . . 26
  - associativity . . . . . 4, 9
    - misuse . . . . . 10
- B**
- backsubstitution . . . . . 51
  - basis . . . . . **118**
    - orthonormal . . . . . 118
  - battleship–captain analogy . . . . . 27
  - Bessel’s inequality . . . . . 75
  - bidiagonal matrix . . . . . 50
  - Bill Gatez . . . . . 25
  - block Cholesky factorization . . . . . 67
  - block diagonal matrix . . . . . 114
  - block LU factorization . . . . . 61
  - block triangular matrix . . . . . 52, 61, 67
    - rank of . . . . . 114
- C**
- canonical vector . . . . . **3**, 7, 13
    - in linear combination . . . . . 5
    - in outer product . . . . . 14
  - captain–battleship analogy . . . . . 27
  - catastrophic cancellation . . . . . 27, **28**
    - example of . . . . . 27
    - occurrence of . . . . . 28
  - Cauchy–Schwarz inequality . . . . . 30
  - Cholesky factorization . . . . . 52, **65**
    - algorithm . . . . . 65
    - generalized . . . . . 67
    - in linear system solution . . . . . 66
    - lower-upper . . . . . 65
    - uniqueness . . . . . 65
    - upper-lower . . . . . 67
  - Cholesky solver . . . . . 66
  - circular shift matrix . . . . . 20
  - closest matrix in the two norm . . . . . 36
  - column pivoting . . . . . 76
  - column space . . . . . **86**, 87
    - and residual . . . . . 93
    - and singular vectors . . . . . 87, 107
    - as a subspace . . . . . 105
    - in partitioned matrix . . . . . 107
    - of full rank matrix . . . . . 88
    - of matrix product . . . . . 106
    - of outer product . . . . . 110
    - of transpose . . . . . 87
    - sum of . . . . . 112
  - column vector . . . . . **1**
  - common digits . . . . . 26
  - commutativity . . . . . 4, 10
    - diagonal matrices . . . . . 22
  - complementary subspace . *see* direct sum
  - complete pivoting . . . . . 58
  - complex . . . . . 1
    - conjugate . . . . . 12
    - multiplication . . . . . 11
    - number . . . . . **12**, 14
  - condition number . . . . . 28
    - bidiagonal matrix . . . . . 50
    - componentwise . . . . . 51
    - least squares . . . . . 96, 99, 100
    - left inverse . . . . . 97
    - linear system . . . . . 45, 47, 48, 50, 51

- matrix addition . . . . . 37
- matrix inversion . . . . . 40–42
- matrix multiplication . . . . . 37,  
38, 55
- normal equations . . . . . 103
- scalar division . . . . . 29
- scalar multiplication . . . . . 28
- scalar subtraction . . . . . 27, 28
- triangular matrix . . . . . 52
- conjugate transpose . . . . . **12**  
inverse of . . . . . 17
  
- D**
- diagonal matrix . . . . . **22**  
commutativity . . . . . 22  
in SVD . . . . . 77
- diagonally dominant matrix . . . . . **42**
- dimension . . . . . **109**  
of left null space . . . . . 109  
of null space . . . . . 109  
of row space . . . . . 109  
uniqueness of . . . . . 109
- dimension formula . . . . . 109
- direct method . . . . . 52  
solution of linear systems  
by . . . . . 52  
stability . . . . . 54, 56, 57
- direct sum . . . . . **115**  
and least squares . . . . . 117  
not unique . . . . . 116  
of matrix columns . . . . . 115  
of subspaces of a matrix . . . . . 115  
unique representation . . . . . 117
- distance to lower rank  
matrices . . . . . 83
- distance to singularity . . . . . 41
- distributivity . . . . . 4, 9
- division . . . . . 29
- dot product . . . . . *see* inner product
  
- F**
- finite precision arithmetic . . . . . xi, 23
- fixed point arithmetic . . . . . xi
- floating point arithmetic . . . . . xi, 26  
IEEE . . . . . 26  
unit roundoff . . . . . 27
- forward elimination . . . . . 51
- Fredholm’s alternatives . . . . . 89
  
- full column rank . . . . . **84**  
direct sum . . . . . 115  
in linear system . . . . . 88  
least squares . . . . . 94  
Moore–Penrose inverse . . . . . 92  
null space . . . . . 88
- full rank . . . . . **84**
- full row rank . . . . . **84**  
column space . . . . . 88  
in linear system . . . . . 88  
Moore–Penrose inverse . . . . . 92
- fundamental theorem of linear  
algebra  
first part . . . . . 109  
second part . . . . . 116
  
- G**
- Gaussian elimination . . . . . 60  
growth in . . . . . 61  
stability . . . . . 60
- generalized Cholesky  
factorization . . . . . 67
- Givens rotation . . . . . **19**  
in QL factorization . . . . . 72  
in QR factorization . . . . . 71, 74  
not a . . . . . 19  
order of . . . . . 70  
position of elements . . . . . 71
- grade point average . . . . . 29
  
- H**
- Hankel matrix . . . . . **3**
- Hermitian matrix . . . . . **15**, 16, 20  
and direct sum . . . . . 117  
column space . . . . . 87  
inverse of . . . . . 18  
null space . . . . . 87  
positive definite . . . . . 63  
test for positive  
definiteness . . . . . 66  
unitary . . . . . 20
- Hermitian positive definite  
matrix . . . . . *see* positive  
definite matrix
- Hilbert matrix . . . . . **3**
- Hölder inequality . . . . . 30
- Householder reflection . . . . . 73

- I**
- idempotent matrix . . . . . **11**
    - and direct sum . . . . . 117
    - and Moore–Penrose inverse . . . . . 94
    - and orthogonal subspaces . . . . . 117
    - column space . . . . . 108
    - norm . . . . . 36, 85
    - null space . . . . . 108
    - product . . . . . 11
    - singular . . . . . 17
    - singular values . . . . . 78
    - subspaces . . . . . 114
  - identity matrix . . . . . **3**, 13, 14, 38, 42
  - IEEE double precision
    - arithmetic . . . . . 26
    - unit roundoff . . . . . 27
  - ill-conditioned linear system . . . . . 24, 25
  - ill-posed least squares problem . . . . . 96
  - imaginary unit . . . . . **12**
  - infinity norm . . . . . **30**, **34**
    - and inner product . . . . . 30
    - in Gaussian elimination . . . . . 60
    - in LU factorization . . . . . 59
    - one norm of transpose . . . . . 36
    - outer product . . . . . 36
    - relations with other norms . . . . . 32, 33, 36
  - inner product . . . . . **5**
    - for polynomial . . . . . 5
    - for sum of scalars . . . . . 5
    - in matrix vector multiplication . . . . . 6, 7
    - properties . . . . . 14
  - intersection of null spaces . . . . . 112
  - intersection of subspaces . . . . . **111**, 112
    - in direct sum . . . . . 115
    - properties . . . . . 114
  - inverse
    - partitioned . . . . . 107
  - inverse of a matrix . . . . . **16**, 17
    - condition number . . . . . 40, 46
    - distance to singularity . . . . . 41
    - left . . . . . 93
      - partitioned . . . . . 17, 18, 107
      - perturbation . . . . . 39
      - perturbed . . . . . 42
      - perturbed identity . . . . . 38, 42
      - residuals . . . . . 41
      - right . . . . . 93
      - singular values . . . . . 78
      - SVD . . . . . 78
      - two norm . . . . . 79
  - invertible matrix . . . . . *see* nonsingular matrix
  - involutory matrix . . . . . 11, **11**, 18, 20
    - inverse of . . . . . 16
- K**
- kernel . . . . . *see* null space
- L**
- LDU factorization . . . . . 61
  - least squares . . . . . **91**
    - and direct sum . . . . . 117
    - angle . . . . . 98, 99
    - condition number . . . . . 96, 99, 100
    - effect of right-hand side . . . . . 96, 99
    - full column rank . . . . . 94
    - ill-conditioned . . . . . 100
    - ill-posed . . . . . 96
    - rank deficient . . . . . 96
    - relative error . . . . . 96, 98, 100, 101
    - residual . . . . . 93
  - least squares residual . . . . . **91**
    - conditioning . . . . . 101
  - least squares solutions . . . . . 91
    - by QR factorization . . . . . 102
    - full column rank . . . . . 94
    - in terms of SVD . . . . . 91, 102
    - infinitely many . . . . . 92
    - minimal norm . . . . . 94
    - Moore–Penrose inverse . . . . . 93
  - left inverse . . . . . **93**
    - condition number . . . . . 97
  - left null space . . . . . **87**
    - and singular vectors . . . . . 87
    - dimension of . . . . . 109
  - left singular vector . . . . . *see* singular vector

- linear combination . . . . . **5, 8**
    - in linear system . . . . . 43
    - in matrix vector
      - multiplication . . . . . 6, 7
  - linear system . . . . . 43
    - condition number . . . . . 45–48, 50, 51
    - effect of right-hand
      - side . . . . . 49, 50
    - full rank matrix . . . . . 88
    - ill-conditioned . . . . . 46
    - nonsingular . . . . . 43
    - perturbed . . . . . 44
    - relative error . . . . . 45, 47, 48, 50, 51
    - residual . . . . . 44
    - residual bound . . . . . 45, 47, 50
    - triangular . . . . . 45
  - linear system solution . . . . . 43, 114
    - by direct method . . . . . 52
    - by Cholesky factorization . . 66
    - by LU factorization . . . . . 60
    - by QR factorization . . . . . 68
    - full rank matrix . . . . . 88
    - nonsingular matrix . . . . . 43
    - what not to do . . . . . 57
  - linearly dependent columns . . . . 73
  - linearly independent columns . . 73
    - test for . . . . . 74
  - lower triangular matrix . . . . . **20**
    - forward elimination . . . . . 51
    - in Cholesky factorization . . 65
    - in QL factorization . . . . . 72
  - lower-upper Cholesky
    - factorization . . . . . 65
  - LU factorization . . . . . 52, **58**
    - algorithm . . . . . 59
    - in linear system solution . . 60
    - permutation matrix . . . . . 59
    - stability . . . . . 60
    - uniqueness . . . . . 21
    - with partial pivoting . . . . 59, 60
- M**
- matrix . . . . . **1**
    - distance to singularity . . . . 41
    - equality . . . . . 8
    - full rank . . . . . 84
    - idempotent . . . . . 114
    - multiplication . . . . . 38
    - nilpotent . . . . . 85, 117
    - normal . . . . . 89, 117
    - notation . . . . . 2
    - powers . . . . . **11, 18**
    - rank . . . . . *see* rank
    - rank deficient . . . . . 84
    - skew-symmetric . . . . . 18
    - square . . . . . **1**
    - vector multiplication . . . . . **6, 9**
      - with linearly dependent
        - columns . . . . . 73
      - with linearly independent
        - columns . . . . . 73
      - with orthonormal
        - columns . . . . . 75, 85
  - matrix addition . . . . . **4**
    - condition number . . . . . 37
  - matrix inversion . . . . . *see* inverse of a matrix
  - matrix multiplication . . . . . **9**
    - condition number . . . . . 37, 38, 46, 55
  - matrix norm . . . . . **33**
    - well-conditioned . . . . . 33
  - matrix product . . . . . **9**
    - column space . . . . . 106
    - condition number . . . . . 37
    - null space . . . . . 106
    - rank of . . . . . 85, 110
  - maximum norm . . . . . *see* infinity norm
  - minimal norm least squares solution
    - 94
  - Moore–Penrose inverse . . . . . **92**
    - column space . . . . . 94
    - defining properties . . . . . 95
    - in idempotent matrix . . . . . 94
    - in least squares . . . . . 93
    - norm . . . . . 94, 95
    - null space . . . . . 94
    - of full rank matrix . . . . . 92
    - of nonsingular matrix . . . . . 94
    - of product . . . . . 95
    - orthonormal columns . . . . . 95
    - outer product . . . . . 94

- partial isometry . . . . . 95
- partitioned . . . . . 95
- QR factorization . . . . . 95
- multiplication . . . . . 28
- multiplier in LU factorization . . . 59
- N**
- nilpotent matrix . . . . . **11**, 18, 85, 117
  - singular . . . . . 18
  - strictly triangular . . . . . 21
- nonsingular matrix . . . . . **16**, 17, 18, 42, 43
  - condition number w.r.t.
    - inversion . . . . . 40
  - distance to singularity . . . . . 41
  - LU factorization of . . . . . 60
  - positive definite . . . . . 63
  - product of . . . . . 18
  - QL factorization of . . . . . 72
  - QR factorization of . . . . . 68
  - triangular . . . . . 21
  - unit triangular . . . . . 21
- norm . . . . . 29, 33
  - defined by matrix . . . . . 33
  - of a matrix . . . . . **33**
  - of a product . . . . . 35
  - of a submatrix . . . . . 35
  - of a vector . . . . . **29**
  - of diagonal matrix . . . . . 36
  - of idempotent matrix . . . . . 36
  - of permutation matrix . . . . . 36
  - reverse triangle
    - inequality . . . . . 32
  - submultiplicative . . . . . 34, 35
  - unit norm . . . . . 30
- normal equations . . . . . **103**
  - instability . . . . . 103
- normal matrix . . . . . **89**, 117
- notation . . . . . 2
- null space . . . . . **86**, 87
  - and singular vectors . . . 87, 108
  - as a subspace . . . . . 105
  - dimension of . . . . . 109
  - in partitioned matrix . . . . . 107
  - intersection of . . . . . 112
  - of full rank matrix . . . . . 88
  - of outer product . . . . . 110
    - of product . . . . . 106
    - of transpose . . . . . 87
- numerical stability . . . . . *see* stability
- O**
- one norm . . . . . **30**, 34
  - and inner product . . . . . 30
  - relations with other
    - norms . . . . . 32, 33, 36
- orthogonal matrix . . . . . **19**
  - in QR factorization . . . . . 21
- orthogonal subspace . . . . . **116**
  - and QR factorization . . . . . 117
  - direct sum . . . . . 117
  - properties . . . . . 117
  - subspaces of a matrix . . . . . 116
- orthonormal basis . . . . . 118
- orthonormal columns . . . . . 75
  - condition number . . . . . 100
  - in polar decomposition . . . . . 85
  - Moore–Penrose inverse . . . . . 95
  - singular values . . . . . 85
- outer product . . . . . **8**, 9, 14
  - column space . . . . . 110
  - in Schur complement . . . . . 59
  - in singular matrix . . . . . 40
  - infinity norm . . . . . 36
  - Moore–Penrose inverse . . . . . 94
  - null space . . . . . 110
  - of singular vectors . . . . . 82
  - rank . . . . . 81, 110
  - SVD . . . . . 81
  - two norm . . . . . 36
- P**
- $p$ -norm . . . . . **30**, **33**
  - with permutation matrix . . . . . 32
- parallelogram equality . . . . . 32
- partial isometry . . . . . 95
- partial pivoting . . . . . 58, 59
- partitioned inverse . . . . . 17, 18, 107
- partitioned matrix . . . . . 20
- permutation matrix . . . . . **19**
  - in LU factorization . . . . . 59
  - partitioned . . . . . 20
  - product of . . . . . 20
  - transpose of . . . . . 20
- perturbation . . . . . 23

- pivot . . . . . 59
- polar decomposition . . . . . **85**  
    closest unitary matrix . . . . . 85
- polar factor . . . . . 85
- polarization identity . . . . . 33
- positive definite matrix . . . . . **63**  
    Cholesky factorization . . . 65, 66  
    diagonal elements of . . . 63, 67  
    generalized Cholesky  
        factorization . . . . . 67  
    in polar decomposition . . . . 85  
    lower-upper Cholesky  
        factorization . . . . . 65  
    nonsingular . . . . . 63  
    off-diagonal elements of . . . 66  
    principal submatrix of . . . . 64  
    Schur complement of . . . . . 64  
    SVD . . . . . 78  
    test for . . . . . 66  
    upper-lower Cholesky  
        factorization . . . . . 67
- positive semidefinite matrix . . . . **63**
- principal submatrix . . . . . **2**  
    positive definite . . . . . 64
- product of singular values . . . . . 80
- Pythagoras theorem . . . . . 32
- Q**
- QL factorization . . . . . 72
- QR factorization . . . . . 52, **68**  
    algorithm . . . . . 71, 74  
    and orthogonal subspace . . 117  
    column space . . . . . 108  
    Moore–Penrose inverse . . . . 95  
    null space . . . . . 108  
    orthonormal basis . . . . . 119  
    rank revealing . . . . . 86  
    thin . . . . . 74, 75  
    uniqueness . . . . . 21, 68  
    with column pivoting . . . . . 76
- QR solver . . . . . 68, 102
- R**
- range . . . . . *see* column space
- rank . . . . . 81  
    and reduced SVD . . . . . 82  
    deficient . . . . . 84  
    full . . . . . 84
- of a submatrix . . . . . 113
- of block diagonal matrix . . 114
- of block triangular  
    matrix . . . . . 114
- of matrix product . . . . . 85, 110
- of outer product . . . . . 81, 110
- of Schur complement . . . . . 114
- of transpose . . . . . 84
- of zero matrix . . . . . 81
- real . . . . . 1
- reduced SVD . . . . . 82
- reflection . . . . . 19  
    Householder . . . . . 73
- relative error . . . . . **26**  
    componentwise . . . . . 31  
    in subtraction . . . . . 28  
    normwise . . . . . 31
- relative perturbation . . . . . 27
- residual . . . . . 44, 91  
    and column space . . . . . 93  
    computation of . . . . . 102  
    large norm . . . . . 97  
    matrix inversion . . . . . 41  
    norm . . . . . 45–47  
    of a linear system . . . . . 44  
    of least squares problem . . . **91**,  
        101  
    relation to perturbations . . . 44  
    small norm . . . . . 45  
    uniqueness . . . . . 93
- residual bound . . . . . 45, 47, 50, 101
- right inverse . . . . . **93**
- right singular vector . . . *see* singular  
    vector
- row space . . . . . **87**  
    and singular vectors . . . . . 87  
    dimension of . . . . . 109
- row vector . . . . . **1**
- S**
- scalar . . . . . **1**
- scalar matrix multiplication . . . . **4**
- Schur complement . . . . . 18, 59, 61  
    positive definite . . . . . 64  
    rank . . . . . 114
- sensitive . . . . . xi, 23

- Sherman–Morrison  
    formula . . . . . 17, 18
- shift matrix . . . . . 13  
    circular . . . . . 20
- singular matrix . . . . . **16**, 17  
    distance to . . . . . 41
- singular value decomposition . . . . . *see*  
    SVD
- singular values . . . . . **77**  
    relation to condition  
        number . . . . . 79  
    conditioning of . . . . . 80  
    extreme . . . . . 79  
    matrix with orthonormal  
        columns . . . . . 85  
    of  $2 \times 2$  matrix . . . . . 78  
    of idempotent matrix . . . . . 78  
    of inverse . . . . . 79  
    of product . . . . . 78, 80  
    of unitary matrix . . . . . 78  
    relation to rank . . . . . 81  
    relation to two norm . . . . . 79  
    uniqueness . . . . . 77
- singular vector matrix . . . . . **77**
- singular vectors . . . . . 81  
    column space . . . . . 107  
    in outer product . . . . . 82  
    in reduced SVD . . . . . 82  
    left . . . . . 81  
    null space . . . . . 108  
    orthonormal basis . . . . . 118  
    relations between . . . . . 82  
    right . . . . . 81
- skew-Hermitian matrix . . . . . 15,  
    **15**, 16
- skew-symmetric matrix . . . . . 15, **15**,  
    16, 18
- stability . . . . . **54**  
    of Cholesky solver . . . . . 66  
    of direct methods . . . . . 54, 56, 57  
    of Gaussian elimination . . . . . 60  
    of QR solver . . . . . 68
- steep function . . . . . 24
- strictly column diagonally dominant  
    matrix . . . . . **42**
- strictly triangular matrix . . . . . 21  
    nilpotent . . . . . 21
- submatrix . . . . . **2**  
    principal . . . . . 2  
    rank of . . . . . 113
- submultiplicative  
    inequality . . . . . 34, 35
- subspace . . . . . **105**  
    complementary . . . . . **115**  
    dimension of . . . . . 109  
    direct sum . . . . . **115**  
    intersection . . . . . **111**  
    orthogonal . . . . . **116**  
    real . . . . . 106  
    sum . . . . . **111**
- subtraction . . . . . 27  
    absolute condition  
        number . . . . . 27  
    absolute error . . . . . 27  
    relative condition number . . . . . 28  
    relative error . . . . . 28
- sum of subspaces . . . . . **111**, 112  
    column space . . . . . 112  
    of a matrix . . . . . 112  
    properties . . . . . 114
- SVD . . . . . **77**  
    of inverse . . . . . 78  
    of positive definite  
        matrix . . . . . 78  
    of transpose . . . . . 77  
    optimality . . . . . 83  
    reduced . . . . . 82  
    singular vectors . . . . . 81
- symmetric matrix . . . . . **15**, 16  
    diagonal . . . . . 22  
    inverse of . . . . . 18
- T**
- thin QR factorization . . . . . 74  
    uniqueness . . . . . 75
- Toeplitz matrix . . . . . **3**, 7
- transpose . . . . . **12**, 85  
    inverse of . . . . . 17  
    of a product . . . . . 13  
    partial isometry . . . . . 95  
    rank . . . . . 84  
    SVD . . . . . 77
- triangle inequality . . . . . 29  
    reverse . . . . . 32

- triangular matrix . . . . . **20**
    - bound on condition
      - number . . . . . 52
    - diagonal . . . . . 22
    - ill-conditioned . . . . . 47
    - in Cholesky
      - factorization . . . . . 65, 67
    - in QL factorization . . . . . 72
    - in QR factorization . . . . . 68
    - linear system solution . . . . . 51
    - nonsingular . . . . . 21
  - two norm . . . . . **30**, 79
    - and inner product . . . . . 30
    - Cauchy–Schwarz
      - inequality . . . . . 30
    - closest matrix . . . . . 36
    - closest unitary matrix . . . . . 85
    - condition number for
      - inversion . . . . . 79
    - in Cholesky solver . . . . . 66
    - in least squares problem . . . . . 91
    - in QR solver . . . . . 68
    - of a unitary matrix . . . . . 36
    - of inverse . . . . . 79
    - of transpose . . . . . 35
    - outer product . . . . . 36
    - parallelogram inequality . . . . . 32
    - polarization identity . . . . . 33
    - relation to singular
      - values . . . . . 79
    - relations with other
      - norms . . . . . 33, 36
    - theorem of Pythagoras . . . . . 32
    - with unitary matrix . . . . . 32
- U**
- UL factorization . . . . . 62
  - uncertainty . . . . . 23, 27, 44
  - unit roundoff . . . . . 27
- unit triangular matrix . . . . . 21
    - in LU factorization . . . . . 21, 58
    - inverse of . . . . . 21
  - unit-norm vector . . . . . 30
  - unitary matrix . . . . . **19**, 41, 85
    - $2 \times 2$  . . . . . 19
    - closest . . . . . 85
    - Givens rotation . . . . . 19
    - Hermitian . . . . . 20
    - Householder reflection . . . . . 73
    - in QL factorization . . . . . 72
    - in QR factorization . . . . . 21, 68
    - in SVD . . . . . 77
    - partitioned . . . . . 20
    - product of . . . . . 20
    - reflection . . . . . 19
    - singular values . . . . . 78
    - transpose of . . . . . 20
    - triangular . . . . . 22
  - upper triangular matrix . . . . . **20**
    - backsubstitution . . . . . 51
    - bound on condition
      - number . . . . . 52
    - in Cholesky
      - factorization . . . . . 65, 67
    - in LU factorization . . . . . 21, 58
    - in QR factorization . . . . . 21, 68
    - linear system solution . . . . . 51
    - nonsingular . . . . . 21
  - upper-lower Cholesky
    - factorization . . . . . 67
- V**
- Vandermonde matrix . . . . . **3**, 8
  - vector norm . . . . . **29**
- Z**
- zero matrix . . . . . **2**