Preface

This book was written for a first-semester graduate course in matrix theory at North Carolina State University. The students come from applied and pure mathematics, all areas of engineering, and operations research. The book is self-contained. The main topics covered in detail are linear system solution, least squares problems, and singular value decomposition.

My objective was to present matrix analysis in the context of numerical computation, with numerical conditioning of problems, and numerical stability of algorithms at the forefront. I tried to present the material at a basic level, but in a mathematically rigorous fashion.

Main Features. This book differs in several regards from other numerical linear algebra textbooks.

- *Systematic development of numerical conditioning*.
  Perturbation theory is used to determine sensitivity of problems as well as numerical stability of algorithms, and the perturbation results built on each other.
  For instance, a condition number for matrix multiplication is used to derive a residual bound for linear system solution (Fact 3.5), as well as a least squares bound for perturbations on the right-hand side (Fact 5.11).
- *No floating point arithmetic*.
  There is hardly any mention of floating point arithmetic, for three main reasons. First, sensitivity of numerical problems is, in general, not caused by arithmetic in finite precision. Second, many special-purpose devices in engineering applications perform fixed point arithmetic. Third, sensitivity is an issue even in symbolic computation, when input data are not known exactly.
- *Numerical stability in exact arithmetic*.
  A simplified concept of numerical stability is introduced to give quantitative intuition, while avoiding tedious roundoff error analyses. The message is that unstable algorithms come about if one decomposes a problem into ill-conditioned subproblems.
  Two bounds for this simpler type of stability are presented for general direct solvers (Facts 3.14 and 3.17). These bounds imply, in turn, stability bounds for solvers based on the following factorizations: LU (Corollary 3.22), Cholesky (Corollary 3.31), and QR (Corollary 3.33).

- *Simple derivations*.
  The existence of a QR factorization for nonsingular matrices is deduced very simply from the existence of a Cholesky factorization (Fact 3.32), without any commitment to a particular algorithm such as Householder or Gram–Schmidt.
  A new intuitive proof is given for the optimality of the singular value decomposition (Fact 4.13), based on the distance of a matrix from singularity.
  I derive many relative perturbation bounds with regard to the perturbed solution, rather than the exact solution. Such bounds have several advantages: They are computable; they give rise to intermediate absolute bounds (which are useful in the context of fixed point arithmetic); and they are easy to derive.
  Especially for full rank least squares problems (Fact 5.14), such a perturbation bound can be derived fast, because it avoids the Moore–Penrose inverse of the perturbed matrix.
- *High-level view of algorithms*.
  Due to widely available high-quality mathematical software for small dense matrices, I believe that it is not necessary anymore to present detailed implementations of direct methods in an introductory graduate text. This frees up time for analyzing the accuracy of the output.
- *Complex arithmetic*.
  Results are presented for complex rather than real matrices, because engineering problems can give rise to complex matrices. Moreover, limiting one's focus to real matrices makes it difficult to switch to complex matrices later on. Many properties that are often taken for granted in the real case no longer hold in the complex case.
- *Exercises*.
  The exercises contain many useful facts. A separate category of easier exercises, labeled with roman numerals, is appropriate for use in class.

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