

# Index

**Boldface** references indicate the pages that should be consulted first in cases where there are several page references for a single entry. These are the pages that give the main idea, or the definition, or the most important fact to be understood about the index entry. If a reference appears alone, however, it is set in ordinary type, even if it is important.

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