

# Preface

This book is devoted to primarily continuous models for a special class of supply chains often called production or supply networks. The aim is to present a mathematical description of different phenomena appearing in planning and managing supply chains. We address both the mathematical modeling as well as techniques for simulation and optimization purposes.

The problem of a continuous description of supply chains and production networks dates back to the early 60's and started with the work of [8, 30]. Significantly, the models were proposed in particular for large volume production on complex networks where a discrete description might fail. Since then, many methods and ideas have been developed concerning the modeling of different features of supply chains, including the efficient simulation and the optimization of product flows among suppliers and customers. In recent years continuous and homogeneous product flow models have been introduced, for example, in [2, 13, 23, 26, 27, 28, 29, 37, 41, 42]. These models have been built in close connection to other transport problems like vehicular traffic flow and queuing theory. Hence, this suggests that the obtained models should be given by partial differential equations for the product flow, similar to those of gas dynamics. Depending on the problem at hand, these equations are possibly accompanied by ordinary differential equations describing the load of inventories. Also some optimization techniques have been proposed in order to answer questions arising in supply chain planning [31, 50, 57].

Starting from a network formulation, we derive equations for a continuous description of homogeneous product flows. The derivation is based on first principles, but the final equations are closely related to discrete event simulations of supply chains. Additionally, we present extensions to include more realistic phenomena. Such extensions consist of systems of partial differential equations or coupled partial and ordinary differential equations. The book surveys the underlying fundamentals and provides evolved mathematical techniques for simulation and efficient optimization of the presented models.

The book is suitable for researchers and students in mathematics, physics, engineering, economics, and related disciplines. Basically, the book could also be used as lecture notes for a course in supply chain theory. Furthermore, interested readers can find many suggestions and open problems for future research.

At first, in Chapter 1, we give some elementary examples which serve as motivation for the modeling approaches following, and then we report mathematical preliminaries about conservation laws and numerical schemes for their discretization. Chapters 2–5 contain a complete and elementary description of the state-of-the-art theory of continuous supply chain models where also similarities and comparisons with already existing models are given. Optimization problems and the corresponding solution techniques are the content of Chapter 6. In particular, the interplay between discrete and continuous optimization

problems is addressed. Numerical discretization issues and computational results in Chapter 7 complete this book.

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