Preface

Why write another textbook on functional analysis and its applications, since there are already many excellent textbooks around?

Apart from the personal pleasure that such an exercise provides to an author, there are other reasons: One, which perhaps constitutes the main originality of this text, was to assemble in a single volume the most basic theorems of linear and of nonlinear functional analysis; another reason was to simultaneously illustrate the wide applicability of these theorems by treating an abundance of applications.

Applications to linear and nonlinear partial differential equations treated here include Korn's inequality and existence theorems in linear elasticity, obstacle problems, the Babuška–Brezzi inf-sup condition, existence theorems for the Stokes and Navier–Stokes equations of fluid mechanics, existence theorems for the von Kármán equations of a nonlinearly elastic plate, and John Ball's existence theorem in nonlinear elasticity. A variety of other applications deals with selected topics from numerical analysis and optimization theory, such as approximation theory, error estimates for polynomial interpolation, numerical linear algebra, basic algorithms of optimization, Newton's method, or finite difference methods.

A special effort has been made to enhance the pedagogical appeal of the book. After Chapter 1, which is essentially a review of results from real analysis and the theory of functions that will be used in the text, self-contained and complete proofs of most of the theorems are provided.[1] These include proofs that are not always easy to locate in the literature, or difficult to reconstitute without an extended knowledge of collateral topics; for instance, self-contained proofs are given of the Poincaré lemma, of the hypoellipticity of the Laplacian, of the existence theorem for Pfaff systems, or of the fundamental theorem of surface theory. Numerous figures and problems (almost 400) have also been included. Historical notes and original references (at least those that I have been able to trace with a reasonable assurance of veracity) have also been included[2] (mostly as footnotes), so as to provide an idea of the genesis of some important results.

It is my belief that this book contains most of the core topics from functional analysis that any analyst interested in linear and nonlinear applications should have encountered at least once in his or her life. More specifically, linear functional analysis and its applications are the subjects of Chapters 2–6, while nonlinear functional analysis and its applications are the subjects of Chapters 7–9.

Of course, choices had to be made, in particular so as to keep the length of the book within reasonable limits. For instance, more specialized topics, such as the Fourier transform,

[1] The symbol $^\flat$ to the left of a theorem indicates one without proof.

[2] With the full knowledge that doing so sometimes constitutes a perilous exercise...

wavelets, spectral theory (save for compact self-adjoint operators), or time-dependent partial differential equations, are not treated.

Several one-semester courses, at the last-year undergraduate or graduate levels, can be taught from this book, such as "Linear Functional Analysis," "Linear and Nonlinear Boundary Value Problems," "Differential Calculus and Applications," "Introduction to Differential Geometry," "Nonlinear Functional Analysis," or "Mathematical Elasticity and Fluid Mechanics." In this respect, it should be easy for an instructor to identify from the table of contents those parts of the book that should be used for any such course. Indeed, I had the pleasure of teaching such courses, primarily at the University Pierre et Marie Curie and at City University of Hong Kong, but also at the University of Texas at Austin, at Cornell University, at Fudan University, at the University of Stuttgart, at l'Ecole Polytechnique Fédérale de Lausanne, at the ETH-Zürich, and at the University of Zürich.

The main prerequisites are a reasonable acquaintance with real analysis, i.e., elementary topology (such as continuity and compactness), the basic properties of metric spaces and Lebesgue integration, and the theory of real-valued functions of one or several real variables. For the reader's convenience, the basic definitions and theorems from these subjects needed in this book are assembled without proofs in the first chapter.

During the writing of this book, I have greatly benefitted from the comments of Liliana Gratie, George Dinca, Cristinel Mardare, Sorin Mardare, and Pascal Azerad, who were kind enough to very carefully read most of the chapters and to suggest numerous significant improvements. Bernard Dacorogna and Vicentiu Radulescu have also provided me with much precious advice. To all of them, my most sincere thanks!

My gratitude is also due to Douglas N. Arnold for his early—and strong—support of the project, and also to Elizabeth Greenspan, Gina Rinelli, and Lisa Briggeman from the Editorial Office of SIAM, with whom it is a real pleasure to cooperate.

Last but not least, I express my deep gratitude and my lasting admiration to my "mathematical heroes" Laurent Schwartz, Richard S. Varga, Jacques-Louis Lions, and Robert Dautray, whose teaching and advice over the years have been invaluable.

I am perfectly aware that, most likely, there are still at places inadequacies, inconsistencies of notations, inadvertently omitted references, or inappropriate attributions of original results. But any adventure (mathematical or otherwise) must come to an end, even if its main protagonist is not fully satisfied with it. Or equivalently, as Paul Halmos said in a much better way, in a pure gem of a paper[3] that any mathematician, pure or applied, should read and reread (I paraphrase him): "The last step for most authors is to stop writing. That's hard."

This is one more reason why I welcome in advance all comments, remarks, criticisms, etc., which should be sent to `mapgc@cityu.edu.hk`, and—who knows—could be used in a second edition.

Hong Kong, November 2012 Philippe G. Ciarlet

[3]P.R. Halmos [1970]: How to write mathematics, *L'Enseignement Mathématique* **16**, 123–152.