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Featured Review: Mathematics & Climate. *By Hans Kaper and Hans Engler.* SIAM, Philadelphia, 2013. $59.00. xx+295 pp., softcover. ISBN 978-1-611972-60-3.

The science of climate change is complex and the field of research is expanding, increasingly drawing in mathematicians and statisticians. Yet the curriculum at the undergraduate level tends to lack material that "introduces students to mathematically interesting topics from climate science" and makes "climate issues understandable to readers coming from fields other than geophysics." This timely contribution from Kaper and Engler fills this gap and provides a textbook for undergraduates in mathematics and statistics wishing to explore climate sciences as an application area.

The writing style is concise and to the point. Complex subjects, such as the role of oceans in climate, atmospheric structure and circulation, and the cryosphere, are dealt with *descriptively*, with very brief but authoritative prose.

Some may say that the book is too ambitious: by dealing with so many complex topics in climate science, from the Earth's energy budget, the ocean conveyor belt, sea-ice shape, the transport of carbon dioxide, and plankton and algae, to El Niño, data inhomogeneity, and extreme events, the authors can only present these topics in a cursory manner. I think, however, that this is intended by the authors. The book is only meant to *expose* students to various interesting topics of current interest in climate science to which mathematical and statistical tools can be profitably applied. By showing how a few conceptual models utilizing only simple mathematics can be used to make sense of complex phenomena, the authors demonstrate the important role that mathematics and mathematicians can play in climate science.

The book's presentation of mathematical theory and techniques is interspersed among the climate topics, sometimes in separate chapters. Some mathematical topics are treated in detail, while others are dealt with briefly and descriptively. Dynamical systems theory has its own chapter and is discussed in detail, in the typical mathematician's definition-and-theorem format. This is followed by a chapter on bifurcation theory for equilibrium solutions of nonlinear differential equations in one and two dimensions. The subsequent two chapters deal with applications: a short chapter on the equilibrium solution of Stommel's two-box model of thermohaline circulation illustrates the possibility of multiple equilibria, bifurcation, and hysteresis. The following short chapter on the three-component Lorenz equations gives an example of an interesting dynamical system, using a numerical solution to show the strange attractors; however, few of the theorems from the dynamical systems chapter are used.

My favorite chapter is Chapter 11, "Fourier Transforms." The chapter presents Fourier series as a trigonometric interpolation and then proceeds to derive the discrete Fourier transform, leading to a discussion of the advantages of the fast Fourier transform (FFT). The FFT is then used to analyze some sample time series, showing that a different perspective can be obtained by examining their power spectra. Correlation and autocorrelation are defined and interpreted. An excellent section on Milankovitch's theory of glacial cycles follows as an application, including a spectral analysis of the forcing function and the observed response.

The chapters on statistical techniques are useful, as they are commonly used to analyze climate time series of observation and model data. Such datasets are nowadays freely available online, and so a student can actually apply the techniques learned to real problems. Chapter 9 very briefly presents regression analysis, which is then applied in Chapter 10 to the carbon dioxide data from Mauna Loa. Chapter 19 presents theorems from statistics that can be used to infer whether or not an observed incidence of extreme events is random. Chapter 20 discusses various data assimilation methods, including the Bayesian approach. The presentation of the statistical results is very brief, which might be due to the perception of the authors that students taking this course probably have already acquired these concepts from other statistics courses.

The long Chapter 14 derives the hydrodynamics equations (partial differential equations) governing fluid flows on a rotating sphere. These are the equations used in general circulation models, which are supposedly the subject of the following chapter on climate models but are not used there. Instead, the authors present arguments for viewing such models as a dynamical system in functional spaces, resulting in an "abstract climate model." No further insight is gained from this viewpoint, other than the fact that when spectrally truncated, the Rayleigh–Bénard convection equations can lead to the three-component Lorenz equations, which form a dynamical system.

Chapter 16, on the El Niño Southern Oscillation (ENSO), returns to the "conceptual models" used so well in the rest of the book to explain the mechanisms behind this quasi-periodic climate pattern that occurs across the equatorial Pacific Ocean. A conceptual model is not derived from the governing partial differential equations, or from first principles, but is instead argued for as being reasonable and plausible. A recharge-oscillator model of Jin and a delayed-oscillator model of Battisti and Hirst are presented, and the former is solved numerically. For the latter, further discussion of Rossby wave and Kelvin wave dispersions is given but does not emphasize their ties to the life cycle of ENSO and its observed period.

Chapter 12, "Zonal Energy Budget," does a thorough job of deriving the energy balance model of the longitudinally averaged Earth. In contrast to the brevity of earlier and later chapters, each topic here is explained in detail, even including a section on the Legendre polynomial expansion of the solutions. As was the case for Chapter 14, this long derivation is also not made use of later. After completing the model by fitting the model parameters to the present climate, the chapter ends. Subsequent chapters never return to the equation derived.

In summary, *Mathematics & Climate* is a delightful short book at the intersection of mathematics and climate science. It serves its purpose well as an excellent textbook for a one-semester course, especially if the instructor has disciplinary knowledge of the topics in climate science.

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