Taylor Polynomial

The Taylor polynomial p_n of degree $\leq n$ of a function f at a point x_0 matches the derivatives up to order n:

$$p_n(x) = \sum_{k=0}^n \frac{f^{(k)}(x_0)}{k!} (x - x_0)^k.$$

The approximation error or remainder can be expressed in the form

$$f(x) - p_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} (x - x_0)^{n+1}$$

with ξ a point between x and x_0 . As a consequence, polynomials of degree $\leq n$ approximate smooth functions on an interval $[x_0 - h, x_0 + h]$ with the order $O(h^{n+1})$.