## Taylor Polynomial

The Taylor polynomial $p_{n}$ of degree $\leq n$ of a function $f$ at a point $x_{0}$ matches the derivatives up to order $n$ :

$$
p_{n}(x)=\sum_{k=0}^{n} \frac{f^{(k)}\left(x_{0}\right)}{k!}\left(x-x_{0}\right)^{k}
$$

The approximation error or remainder can be expressed in the form

$$
f(x)-p_{n}(x)=\frac{f^{(n+1)}(\xi)}{(n+1)!}\left(x-x_{0}\right)^{n+1}
$$

with $\xi$ a point between $x$ and $x_{0}$. As a consequence, polynomials of degree $\leq n$ approximate smooth functions on an interval $\left[x_{0}-h, x_{0}+h\right]$ with the order $O\left(h^{n+1}\right)$.

