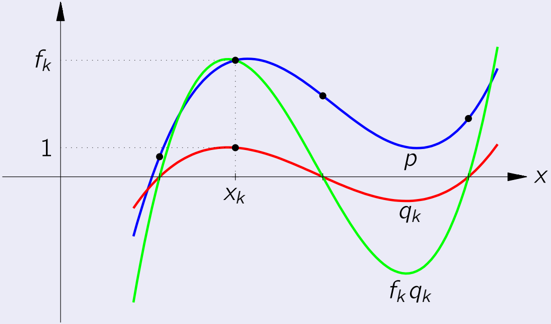
Lagrange Form of Interpolating Polynomials

Any values f_k at $n+1$ distinct points x_k can be interpolated uniquely by a polynomial p of degree $\leq n$:

$$p(x_k) = f_k, \quad k = 0, \ldots, n\,.$$



The Lagrange form of the interpolant is

$$p(x) = \sum_{k=0}^{n} f_k \, q_k(x), \quad q_k(x) = \prod_{\ell \neq k} \frac{x - x_\ell}{x_k - x_\ell}\,,$$

with Lagrange polynomials q_k, which are equal to 1 at x_k and vanish at all other interpolation points x_ℓ , $\ell \neq k$.

Aitken-Neville Scheme

If p_k^{m-1} interpolates f at distinct points $x_k, \dots, x_{k+m-1}$, then

$$p_k^m = (1 - w_k^m) p_k^{m-1} + w_k^m p_{k+1}^{m-1}, \quad w_k^m(x) = \frac{x - x_k}{x_{k+m} - x_k},$$

interpolates at $x_k, \dots, x_{k+m}$.

Starting with $p_k^0 = f(x_k)$, we obtain interpolating polynomials of successively higher degree with a triangular scheme:

$$
\begin{array}{ccccccc}
p_0^0 & \rightarrow & p_0^1 & \rightarrow & p_0^2 & \cdots \rightarrow & p_0^n \\
p_1^0 & \overset{\nearrow}{\rightarrow} & p_1^2 & \nearrow & & & \\
p_2^0 & \nearrow & & & & & \\
\vdots & & & & & & \\
p_n^0 & & & & & &
\end{array}
$$

where the arrows $\rightarrow$ and $\nearrow$ pointing to p_k^m indicate multiplication with $(1 - w_k^m)$ and w_k^m, respectively.

Starting with $p_k^0 = f(x_k)$, we obtain interpolating polynomials of successively higher degree with a triangular scheme:

$$
\begin{array}{ccccccc}
p_0^0 & \rightarrow & p_0^1 & \rightarrow & p_0^2 & \cdots \rightarrow & p_0^n \\
p_1^0 & \overset{\nearrow}{\rightarrow} & p_1^2 & \nearrow & & & \\
p_2^0 & \nearrow & & & & & \\
\vdots & & & & & & \\
p_n^0 & & & & & &
\end{array}
$$

where the arrows $\rightarrow$ and $\nearrow$ pointing to p_k^m indicate multiplication with $(1 - w_k^m)$ and w_k^m, respectively. The final polynomial p_0^n has degree $\leq n$ and interpolates at $x_0, \ldots, x_n$.