Bernstein Polynomials

The Bernstein polynomials of degree n are defined by

$$b_k^n(x) = \binom{n}{k}(1-x)^{n-k}x^k, \quad k = 0, \dots, n\,.$$

They form a basis for the space $\mathbb{P}^n$ of polynomials of degree $\leq n$ which is symmetric with respect to the standard parameter interval $[0, 1]$. In particular, for $j, k \in \{0, \dots, n\}$,

$$x^j = \sum_{k=j}^{n} \binom{k}{j} \Big/ \binom{n}{j}\, b_k^n(x), \quad b_k^n(x) = \sum_{j=0}^{n-k} (-1)^j \binom{n}{k}\binom{n-k}{j} x^{j+k}\,,$$

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which describes the conversion between monomial and Bernstein form. In identities and recursions involving Bernstein polynomials it is often convenient to use indices $k \in \mathbb{Z}$ outside the range $\{0, \ldots, n\}$. For such k, $b_k^n(x)$ is set to zero, in agreement with a similar convention for binomial coefficients.