Properties of Bernstein Polynomials

The Bernstein polynomials of degree n are nonnegative on the standard parameter interval $[0,1]$ and sum to one:

$$\sum_{k=0}^{n} b_k^n(x) = 1\,.$$

Moreover, b_k^n has a unique maximum at $x = \frac{k}{n}$ on $[0,1]$.

Properties of Bernstein Polynomials

The Bernstein polynomials of degree n are nonnegative on the standard parameter interval $[0, 1]$ and sum to one:

$$\sum_{k=0}^{n} b_k^n(x) = 1\,.$$

Moreover, b_k^n has a unique maximum at $x = \frac{k}{n}$ on $[0, 1]$.
At the interval endpoints 0 and 1, only the first and the last Bernstein polynomials are nonzero, respectively:

$$\begin{aligned} b_0^n(0) = 1, \quad b_1^n(0) = \cdots = b_n^n(0) = 0, \\ b_0^n(1) = \cdots = b_{n-1}^n(1) = 0, \quad b_n^n(1) = 1\,. \end{aligned}$$

As a consequence, a polynomial in Bernstein form, $p = \sum_{k=0}^{n} c_k b_k^n$, is equal to c_0 at $x = 0$ and equal to c_n at $x = 1$. This property is referred to as endpoint interpolation.

Identities for Bernstein Polynomials

The Bernstein polynomials b_k^n, $k = 0, \ldots, n$, satisfy the following identities.

Symmetry:

$$b_k^n(1 - x) = b_{n-k}^n(x)\,.$$

Identities for Bernstein Polynomials

The Bernstein polynomials b_k^n, $k = 0, \ldots, n$, satisfy the following identities.

Symmetry:

$$b_k^n(1-x) = b_{n-k}^n(x)\,.$$

Recursion:

$$b_k^n(x) = x\, b_{k-1}^{n-1}(x) + (1-x)\, b_k^{n-1}(x)\,.$$

Identities for Bernstein Polynomials

The Bernstein polynomials b_k^n, $k = 0, \ldots, n$, satisfy the following identities.

Symmetry:

$$b_k^n(1-x) = b_{n-k}^n(x)\,.$$

Recursion:

$$b_k^n(x) = x\, b_{k-1}^{n-1}(x) + (1-x)\, b_k^{n-1}(x)\,.$$

Differentiation:

$$(b_k^n)' = n\left(b_{k-1}^{n-1} - b_k^{n-1}\right)\,.$$

Identities for Bernstein Polynomials

The Bernstein polynomials b_k^n, $k = 0, \ldots, n$, satisfy the following identities.

Symmetry:

$$b_k^n(1-x) = b_{n-k}^n(x)\,.$$

Recursion:

$$b_k^n(x) = x\, b_{k-1}^{n-1}(x) + (1-x)\, b_k^{n-1}(x)\,.$$

Differentiation:

$$(b_k^n)' = n\left(b_{k-1}^{n-1} - b_k^{n-1}\right)\,.$$

Integration:

$$\int_0^1 b_k^n = \frac{1}{n+1}\,.$$

Identities for Bernstein Polynomials

The Bernstein polynomials b_k^n, $k = 0, \ldots, n$, satisfy the following identities.

Symmetry:

$$b_k^n(1-x) = b_{n-k}^n(x)\,.$$

Recursion:

$$b_k^n(x) = x\, b_{k-1}^{n-1}(x) + (1-x)\, b_k^{n-1}(x)\,.$$

Differentiation:

$$(b_k^n)' = n\left(b_{k-1}^{n-1} - b_k^{n-1}\right)\,.$$

Integration:

$$\int_0^1 b_k^n = \frac{1}{n+1}\,.$$

We note that $b_{-1}^{n-1} = b_n^{n-1} = 0$ in the second and third identities, according to the standard convention.