Differentiation

The parametrization

$$p = \sum_{k=0}^{n} c_k \, b_k^n$$

of a Bézier curve is differentiated by forming differences of the control points:

$$p' = n \sum_{k=0}^{n-1} (\Delta c_k) \, b_k^{n-1}$$

with $\Delta c_k = c_{k+1} - c_k$.

More generally, the mth derivative parametrizes a Bézier curve of degree $\leq n - m$ with control points

$$\frac{n!}{(n-m)!}\,\Delta^m c_k, \quad k = 0, \ldots, n-m.$$

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In particular,

$$\binom{n}{m}\Delta^m c_0, \binom{n}{m}\Delta^m c_{n-m}, \quad m = 0, \ldots, n,$$

are the Taylor coefficients at the endpoints.