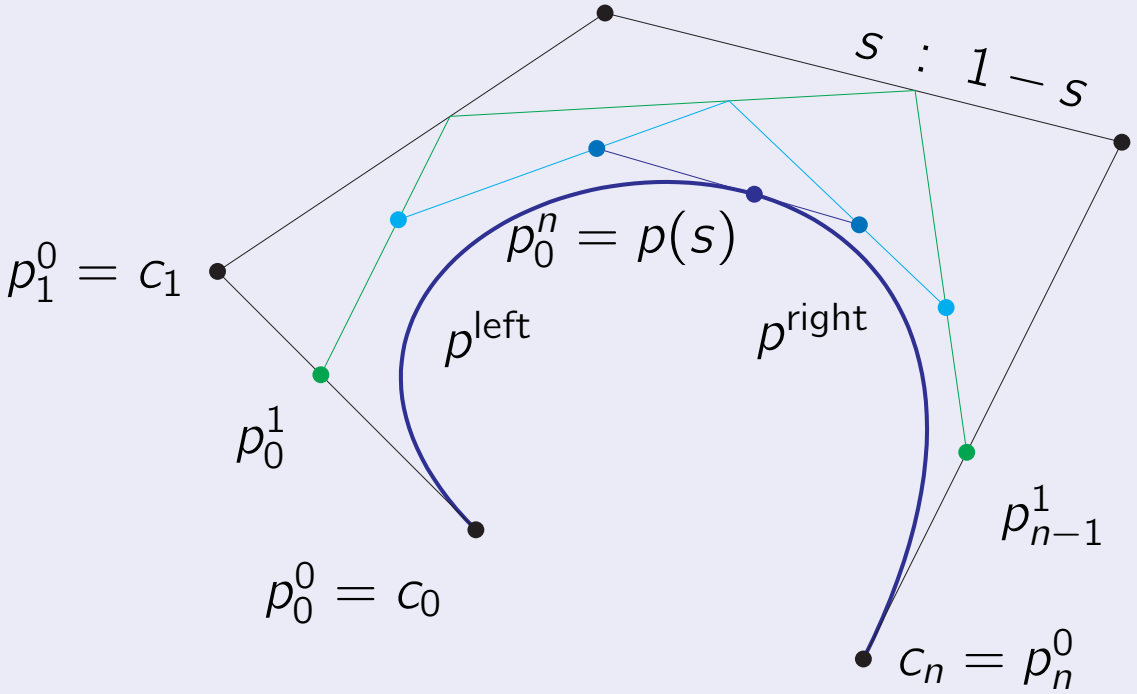
Subdivision

A Bézier curve parametrized by

$$p(t) = \sum_{k=0}^{n} c_k \, b_k^n(t), \quad 0 \le t \le 1\,,$$

can be split into two parts corresponding to the subintervals $[0, s]$ and $[s, 1]$ with the aid of de Casteljau's algorithm.



The first and last control points, p_0^m and p_{n-m}^m, generated with the mth de Casteljau step yield the control points of the left and right curve segments:

$$\begin{aligned} p^{\text{left}}(t) = p(st) &= \sum_{m=0}^{n} p_0^m \, b_m^n(t), \\ p^{\text{right}}(t) = p(s + (1-s)t) &= \sum_{m=0}^{n} p_m^{n-m} \, b_m^n(t). \end{aligned}$$

Hence, the left and right control points correspond to the first and last diagonal of de Casteljau's scheme.

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Hence, the left and right control points correspond to the first and last diagonal of de Casteljau's scheme.

$$\begin{tikzcd}
p_0^0 \arrow[dr] & & p_1^0 \arrow[dl] \arrow[dr] & \cdots \arrow[dl] & p_{n-1}^0 \arrow[dr] & & p_n^0 \arrow[dl] \\
& p_0^1 \arrow[dr] & & \cdots & & p_{n-1}^1 \arrow[dl] & \\
& & \ddots \arrow[dr, "1-s"'] & \vdots & \iddots \arrow[dl, "s"] & & \\
& & & p_0^n & & &
\end{tikzcd}$$