

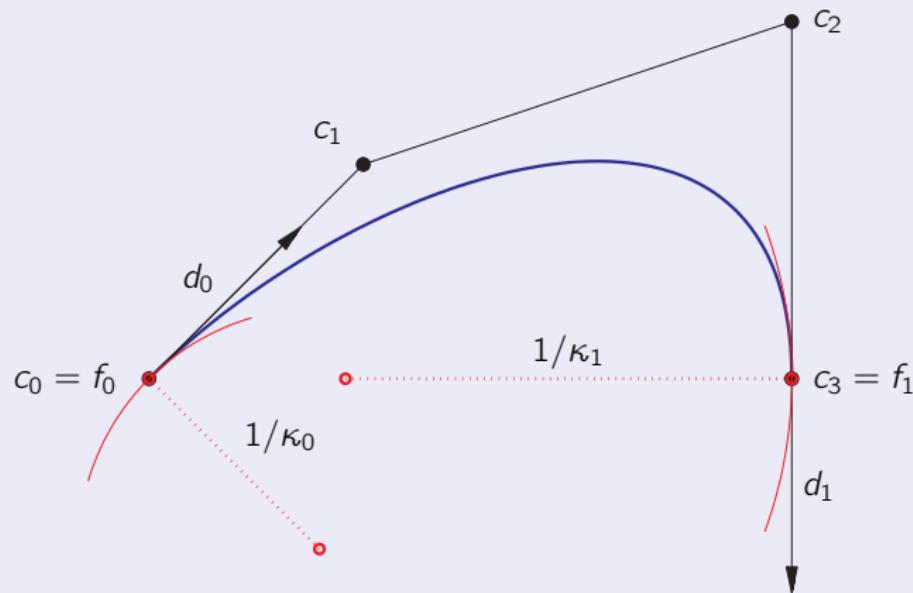
Geometric Hermite Interpolation

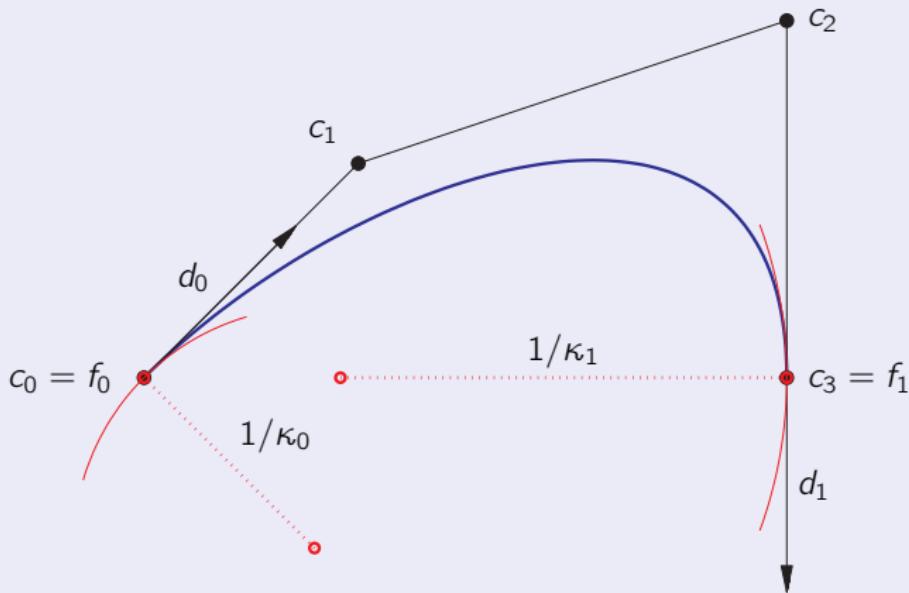
The control points c_0, \dots, c_3 of a planar cubic Bézier curve which interpolates the points f_j , the unit tangent directions d_j , and the signed curvatures κ_j ($j = 0, 1$) at the endpoints $t = 0, 1$ of the parameter interval satisfy

$$c_0 = f_0, \quad c_3 = f_1, \quad c_1 = f_0 + \alpha_0 d_0 / 3, \quad c_2 = f_1 - \alpha_1 d_1 / 3.$$

The lengths α_j of the tangent vectors are positive solutions of the nonlinear system

$$\begin{aligned}\kappa_0 \alpha_0^2 &= \det(d_0, 6(f_1 - f_0) - 2\alpha_1 d_1), \\ \kappa_1 \alpha_1^2 &= \det(d_1, 2\alpha_0 d_0 - 6(f_1 - f_0)).\end{aligned}$$





If the data correspond to a smooth curve with nonvanishing curvature, then for a sufficiently small distance $|f_1 - f_0|$, a solution of the nonlinear system exists and the error of the cubic Bézier approximation is of order $\mathcal{O}(|f_1 - f_0|^6)$.