Properties of a Rational Bézier Curve

A rational Bézier curve parametrized by

$$r(t) = \frac{\sum_{k=0}^{n} (c_k w_k)\, b_k^n(t)}{\sum_{k=0}^{n} w_k\, b_k^n(t)}, \quad t \in [0,1]\,,$$

has the following basic properties:

- $r(t)$ lies in the convex hull of the control points $c_0, \ldots, c_n$,
- $\lim_{w_k \to \infty} r(t) = c_k$ for $t \in (0,1)$,
- $r(0) = c_0$, $r(1) = c_n$,
- $r'(0) = n \frac{w_1}{w_0} (c_1 - c_0)$, $r'(1) = n \frac{w_{n-1}}{w_n} (c_n - c_{n-1})$.

Parameter Transformation and Scaling

A rational Bézier curve parametrized by $\sum_k (c_k w_k)\, b_k^n / \sum_k w_k\, b_k^n$, is left invariant by scaling of the weights

$$w \to \lambda w$$

and by a linear rational parameter transformation of the form

$$t = \frac{s}{\varrho s + 1 - \varrho}\,, \quad \varrho < 1\,.$$

The two degrees of freedom can be used to change the first and last weight to 1,

$$w_k \to \tilde{w}_k = w_0^{k/n-1} w_n^{-k/n}\, w_k\,,$$

which is referred to as standard parametrization of a rational Bézier curve.