Bézier Form of Conic Sections

Any rational quadratic Bézier curve parametrized by

$$r = \frac{(c_0 w_0)b_0^2 + (c_1 w_1)b_1^2 + (c_2 w_2)b_2^2}{w_0 b_0^2 + w_1 b_1^2 + w_2 b_2^2}$$

represents a segment of a conic section.

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Conversely, any nondegenerate conic section can be represented by an extended parametrization r(t), $t \in \mathbb{R} \cup \{\infty\}$.



As is indicated in the figure, if the control points are not collinear, the type of the rational quadratic Bézier curve corresponds to the sign of $d = w_0 w_2 - w_1^2$.